The Expected Real Return to Equity

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Abstract

The expected return to equity is generally computed as the historical average of realized returns. This study presents an alternate, forward-looking estimate of the expected return using a production-based framework. The underlying models maps dividends, earnings, and investment opportunities onto the market value of equity. The estimates reveal an expected annual real return to equity of 5% from the mid 80's onward. The estimated expected return declines by about 2.5% from the twenty year period prior to 1987 to the period afterward. These findings suggest that either there was a structural break in expected returns or that expected returns vary slowly over long horizons. In addition, the theoretical analysis demonstrates the necessity of incorporating growth in asset pricing models. Some studies ignore growth, resulting in a significant calibration error for both the real return to equity and the risk-free rate.

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1 Introduction

The expected return to the aggregate stock market is a key variable in the decisions of both individual investors and corporations, as emphasized by Merton (1980). A sample average of realized returns provides the simplest, and most widely used, estimate of expected returns. However, this approach ignores the possibility of either structural breaks in expected returns or changes over long horizons. Motivated by this concern, a recent literature has examined various forward looking measures of expected returns based primarily on analysts forecasts or investor surveys.[1] This study presents an alternate model-based estimation of the expected return to equity.

The intuition underlying the estimation is that the expected return to equity provides a mapping between the current dividends, investment opportunities, and valuation of a firm. Holding all else constant, a higher expected return translates to a lower market value. Thus, one could use data on the dividends, earnings, and valuations of the US stock market to infer the expected returns to equity given a model that captures the dynamics of these variables.

The model underlying the estimation is a variant of the standard production-based asset pricing model employed by Cochrane (1991), Cochrane (1996), Jermann (1998), Gomes, Kogan, and Zhang (2003), Kogan (2004), Zhang (2005), Gomes, Yaron, and Zhang (2006), and others. The model prices an aggregate equity claim, compared to the per-share claim typically priced in the literature.[2] The aggregate firm uses its assets and labor to produce output and earn profits. The firm finances interest payments, taxes, and investment from its profits and pays out the remaining cash as a dividend. The model thus incorporates both taxes and leverage into the relation between the earnings process and dividends process. Growth opportunities arise in the model as the firm varies its optimal scale following persistent shocks to profitability.

I estimate the parameters of this model on data for a U.S. representative firm constructed by aggregating firm level data from Compustat.[3] I perform the analysis on two samples: the first uses quarterly Compustat data from 1984/1/1 to 2008/06/30; the second sample uses annual Compustat data from 1967/1/1 to 2007/12/31. Data availability on dividends and stockholders' equity constrain the start dates of the two samples, respectively. Both samples exclude financial

[1] See Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2001), Claus and Thomas (2001), Graham and Harvey (2005), and Pastor, Sinha, and Swaminathan (2008).

[2] Bansal and Yaron (2007) emphasize this distinction and compare the implications of pricing an aggregate equity claim versus a per-share claim. Larrain and Yogo (2008) examine the present value relationship between asset prices and payouts using data on the aggregate stock market.

[3] Working with a representative firm simplifies the analysis by abstracting from idiosyncratic firm level shocks, which may be relevant for understanding the cross-section of asset prices. McGrattan and Prescott (2005) use the representative firm constructed in the Flow of Funds accounts for their analysis.

firms and regulated utilities.

The estimates using quarterly data imply a mean expected annual real return to equity of about 5% over the sample period. Lowering the assumed trend economic growth rate from 3% to 2% reduces the mean expected return to about 4%. These estimates are within the range of values for expected returns obtained by Claus and Thomas (2001), Fama and French (2002), and Graham and Harvey (2005).[4] The results suggest that mean expected returns to the aggregate stock market over the past twenty-five years have been lower than historical returns, as argued by Lettau, Ludvigson, and Wachter (2008) and Cogley and Sargent (2008). The findings also support the decision in the limited participation literature to use a lower return to equity than would be indicated by historical returns (see Gomes and Michaelides (2005), Polkovnichenko (2007), and Gust and Lopez-Salido (2009)).

The estimates obtained using annual data also provide evidence for a decline in expected returns. I find a mean expected return of about 6% from 1967 to 2007, higher than the estimate obtained using quarterly data. Splitting the sample into two equal periods, I find mean expected returns of 8% for the twenty years prior to 1987 and only 5.5% for the subsequent twenty. This decline is economically quite significant.

The gap between the estimated expected return to equity and realized returns may reflect a structural break in the economy or a slow moving change in expected returns.[5] Lettau, Ludvigson, and Wachter (2008) argue that the decline in equity premium arose from a reduction in macroeconomic volatility. Cogley and Sargent (2008) attribute the decline in the equity premium to slow moving changes in risk attitudes following the Great Depression. Bansal and Yaron (2004) argue that slow moving changes in consumption growth drives asset prices. Panageas and Yu (2006) present a model in which technological change drives slow moving changes in expected returns over long horizons. Increased participation in the stock market may also lower the expected returns to equity. In contrast, Veronesi and Pastor (2008) provide an alternate view that the increase in equity values was a temporary phenomenon associated with the introduction and adoption of the Internet. While the results in this study can not distinguish between these explanations, it suggests that investors should not simply return on historical returns to equity in forming portfolio and savings choices.

The method yields a surprisingly precise estimate of the expected return to equity. The precision

[4] The estimated expected returns are higher than the estimates of Blanchard (1993) and Jagannathan, McGrattan, and Scherbina (2001).

[5] See Pastor and Stambaugh (2001) and Kim, Morley, and Nelson (2005) for a Bayesian approach to structural breaks in equity returns.

of the estimate arises from the sensitivity of the optimal policies and the value function to changes in mean expected returns. A small reduction in expected returns increases the present value of all future dividends. This translates to a substantial impact on firm value. Thus, given data on market-to-book values and dividends, the model yields a fairly precise estimate of the mean expected return over the sample period.

The findings indicate that investors expect future returns to be lower, on average, than historical returns. This has sharp implications for the investment decisions of both individual and institutional investors. Current asset allocation advice is mostly based on properties of historical returns. A lower expected return to equity implies that individuals need to save more to fund retirement expenses and possibly reduce their allocation to equities in their portfolios. A lower expected return also impacts the actuarial calculation of pension funds and insurance firms, who base their decisions on annual expected nominal returns of 6% to 8%.

The development of the model employed in the estimation also yields an important theoretical result on translating models with growth into stationary models without growth. A growing literature uses production based models to study asset prices. Many of these studies employ stationary models without growth and assume that their results map directly onto the observed data.[6] However, I prove that the return to equity in the stationary model without growth equals the return in the model with growth minus the growth rate. The intuition for this result is quite simple: the return to equity in the model with growth includes a component that reflects economic growth that would be absent in the stationary model without growth. This implies that the correctly calibrated stationary model should have a return to equity equal to the observed data value minus the growth rate. The difference is economically quite substantial. A calibration targeting a mean annual return to equity of 6.5% in a model without growth yields an actual return of 9.5% when translated to an economy with a 3% growth rate, as would be necessary for comparing with the observed data. A similar argument applies for the calibration of the risk-free rate.

This study is organized as follows. Section 2 derives the model relating earnings, dividends, and valuations and maps the stationary model onto an economy with growth. Section 3 discusses the data used in the estimation and the identification of the model parameters. Section 4 presents the results. Section 5 discusses some implications of the findings and Section 6 concludes.

[6] A partial listing of such studies include: Carlson, Fisher, and Giammarino (2004), Guvenen (2008), Zhang (2005), Livdan, Sapriza, and Zhang (2008), and Gomes and Schmid (2008).

2 Model

Consider a representative agent economy with a representative firm that employs capital and labor to produce output. The representative agent has a time-separable per period utility function. The agent provides a fixed supply of labor at a wage rate determined by market clearing conditions in the labor market. The agent also invests in the representative firm.

The representative firm produces has a Cobb-Douglas production function that uses assets and labor as inputs. The firm faces a downward sloping demanding curve. Let X_t denote a level of labor augmenting technology that grows at a constant rate γ. This drives all economic growth in the model. Given these assumptions, Appendix A derives the profit function of the firm, $\tilde{\Pi}(K_t, X_t, z_t)$, under balanced growth conditions.

$$\tilde{\Pi}(K_t, X_t, z_t) = z_t K_t^{\theta} X_t^{1-\theta} - cX_t, \tag{1}$$

where K_t denotes total assets, z_t denotes the productivity level, and c denotes the per period fix cost of operations. The trend growth in the fixed cost ensures that the firm does not outgrow the fixed cost over time.

The productivity level, z_t, measures deviations from trend growth levels and is assumed to follow an auto-regressive process with

$$\begin{aligned} \log(z_{t+1}) &= \mu + \rho \log(z_t) + \epsilon_t, \\ \epsilon_t &\sim N(0, \sigma), \end{aligned} \tag{2}$$

where ϵ_t denotes shocks to aggregate productivity. Eberly, Rebelo, and Vincent (2008) begin with the above profit function. Appendix A develops it from a more fundamental basis, and also characterizes the conditions necessary to obtain such a profit function. The analysis also demonstrates that the curvature of the profit function is a function of both the capital share, α, and the elasticity of demand, ν.[7] The literature tends to primarily describe this curvature only in terms of the one or the other parameter. The analysis highlights that the appropriate curvature parameter may vary with the specification for technological progress.

[7]The specific functional form depends on the assumption on the whether technological change was labor or capital augmenting. Assuming that the impact of technological progress entered into the economy through the labor supply, one obtains $\theta = \alpha(1 - \nu)$.

The firm is financed through both borrowing and equity. Denote the level of borrowing by B_t. The main purpose of including borrowing in the model is to incorporate leverage into the relationship between the earning and valuation processes of the firm. For simplicity, the borrowing is assumed to be riskless with an interest rate of r. Incorporating risky debt would require both modelling a cross-section of heterogenous firms and the pricing of defaultable debt (see Hennessy and Whited (2007)).

The firm funds physical investment from its cash flow. Denote investment in physical capital by I_t. The capital accumulation equation is given by

$$K_{t+1} = K_t(1-\delta) + I_t,$$

where δ equals the depreciation rate. The aggregate firm does not face any additional costs of disinvestment as in Abel and Eberly (1996). The firm also faces a quadratic adjustment cost of investment given by $\lambda \frac{I_t^2}{K_t}$. As the model does not contain any financial frictions, the investment friction functions as the only damping device for the firm's policies. The absence of financial frictions are primarily for simplicity, as they are unlikely to play an influential role in the relationship between the earnings and valuations of the aggregate firm.

The firm uses its cash flow to fund investment, pay interest to creditors, pay taxes to the government, and pay dividends to shareholders. The taxes are paid on income adjusted for depreciation and interest expenses, as in the tax code. For simplicity, I consider a linear tax rate and calibrate the tax rate, τ, using data on aggregate taxes and aggregate pre-tax income accruing to shareholders. Given these assumptions, the dividend payout of the firm, $D(K_t, X_t, z_t)$, is given by

$$D(K_t, X_t, z_t) = \tilde{\Pi}(K_t, X_t, z_t)(1-\tau) - B_t(1 + r(1-\tau)) + B_{t+1} + \delta K_t \tau - I_t - \frac{\lambda I_t^2}{2K_t}. \quad (3)$$

The dividend paid by the firm depends on its after tax profits, investment costs, and the change in borrowing. The tax deductability of depreciation also increases the potential dividend payout of the firm.

Let $M_{t,t+1}$ denote the pricing kernel of the economy, which we will subsequently parametrize. The value of the firm, $V(K_t, X_t, z_t)$, can be expressed as the solution to the following Bellman equation:

$$
\begin{aligned}
V(K_t, X_t, z_t) &= \max_{K_{t+1}, I_t} D(K_t, X_t, z_t) + E\left[M_{t,t+1} V(K_{t+1}, X_{t+1}, z_{t+1})\right] \qquad (4) \\
K_{t+1} &= K_t(1-\delta) + I_t \\
X_{t+1} &= X_t(1+\gamma).
\end{aligned}
$$

Note that X_t denotes the current technology level that grows over time at a steady rate. One can then map the above value function onto an economy without growth by dividing by X_t. This detrends the value function and translate it into one obtained from a stationary economy. This mapping highlights the key adjustments necessary in mapping results from stationary models onto observed data.

Denote the detrended variables by lowercase letters. Thus,

$$
k_t = \frac{K_t}{X_t},\ i_t = \frac{I_t}{X_t},\ \tilde{\pi}_t = \frac{\tilde{\Pi}_t}{X_t},\ b_t = \frac{B_t}{X_t},\ d_t = \frac{D_t}{X_t}
$$

and

$$
v(k_t, z_t) = \frac{V(K_t, X_t, z_t)}{X_t}.
$$

It is fairly straightforward to derive that the detrended value of the firm is given by the following Bellman equation:

$$
\begin{aligned}
v(k_t, z_t) &= \max_{k_{t+1}, i_t} d(k_t, z_t) + E\left[M_{t,t+1}(1+\gamma) v(k_{t+1}, z_{t+1})\right] \qquad (5) \\
d(K_t, z_t) &= \tilde{\pi}(k_t, z_t)(1-\tau) - b_t(1 + r(1-\tau)) + b_{t+1}(1+\gamma) + \delta k_t \tau - i_t - \frac{\lambda i_t^2}{2k_t}. \\
(1+\gamma)k_{t+1} &= k_t(1-\delta) + i_t.
\end{aligned}
$$

Observe that the terms involving $t+1$ are now multiplied by $(1+\gamma)$. This arises due to the fact that when detrending $t+1$ variables by X_t, one needs to include a $\frac{X_{t+1}}{X_t} = 1+\gamma$ term in order to detrended $t+1$ values by X_{t+1}.[8] More importantly, this implies that the transformation of the economy into a stationary form involves an adjustment in the pricing kernel to take into account economic growth.

[8]Algebraically, $\frac{K_{t+1}}{X_t} = \frac{K_{t+1}}{X_{t+1}} \frac{X_{t+1}}{X_t} = k_{t+1}(1+\gamma)$.

2.1 Estimation of the expected return to equity

The above model can be employed to provide an estimate of the expected return to equity. Fundamentally, we can employ data on income accruing to shareholders, dividends, and firm value to estimate the structural parameters of the above model given a parametrization of the pricing kernel. Appendices C and D detail the simulated method of moments estimator used in the study.[9] The intuition behind the estimation is that the expected return transforms income data into valuation data, and given data on incomes and valuations, we can infer the expected return.

This analysis provides an alternate, forward looking perspective on the equity premium that may differ from results based on historical data. Fama and French (2002) present the results of a related analysis that argues dividend growth models suggest that the true real return to equities may be lower than historical values. Claus and Thomas (2001) use analysts forecasts to argue that the equity premium is much lower than historical values. While the hypothesis that future returns should simply reflect past returns is a compelling null hypothesis there are robust arguments to the contrary. Stock market participation has increased steadily over the past century, alongside a decrease in aggregate economic risk. Cogley and Sargent (2008) argue that the Great Depression lead to an increase in risk aversion that has slowly dissipated over time. Lettau, Ludvigson, and Wachter (2008) argue that declining macroeconomic risk has lead to a decline in the expected equity premium. These arguments motivate a forward looking estimation of the expected return to equity.

2.2 Mapping models without economic growth to data

The above analysis also enables the derivation of a key result mapping returns in the stationary economy to those in the economy with growth, which corresponds to the observed data. Let $R_{t,t+1}, \tilde{R}_{t,t+1}$ denote the return to the firm in the growth economy and detrended economy, respectively. Thus,

$$\begin{aligned} R_{t,t+1} &= \frac{V(K_{t+1}, X_{t+1}, z_{t+1})}{V(K_t, X_t, z_t) - D(K_t, X_t, z_t)} \\ \tilde{R}_{t,t+1} &= \frac{v(K_{t+1}, z_{t+1})}{v(K_t, z_t) - d(K_t, z_t)}. \end{aligned} \tag{6}$$

[9] Other studies that use this estimation method include Hennessy and Whited (2005), Cooper and Haltiwanger (2006), Hennessy and Whited (2007), Bloom (2008), Eberly, Rebelo, and Vincent (2008) and Kogan, Livdan, and Yaron (2008).

Now, we can establish the following:

Proposition 1 *The log returns to the firm in the stationary economy equals the log returns in the economy with growth minus the logarithm of one plus the growth rate.*

Proof. The proof follows from a straightforward transformation of the return in the growth economy to the corresponding terms in the stationary economy.

$$\begin{aligned}
\log R_{t,t+1} &= \log V(K_{t+1}, X_{t+1}, z_{t+1}) - \log(V(K_t, X_t, z_t) - D(K_t, X_t, z_t)) \\
&= \log \frac{V(K_{t+1}, X_{t+1}, z_{t+1})}{X_{t+1}} - \log \frac{(V(K_t, X_t, z_t) - D(K_t, X_t, z_t))}{X_t} + \log \frac{X_{t+1}}{X_t} \\
&= \log v(k_{t+1}, z_{t+1}) - \log(v(k_t, z_t) - d(k_t, z_t)) + \log(1+\gamma) \\
&= \log \tilde{R}_{t,t+1} + \log(1+\gamma).
\end{aligned}$$

■

As the growth rate are quite small, one could also approximate the $\log(1+\gamma)$ term with γ. The above proof employs the derivation of the value function of the firm, which may be restrictive. Appendix B demonstrates the same result in a more general manner using the representative agent's budget constraint.

The above analysis demonstrates the need for adjusting the returns obtained in stationary economies for economic growth when we wish to map them into the data. The intuition for this result is quite simple: the realized returns in an economy with growth include a component that arises from the presence of growth; returns in a corresponding economy without growth equal the true returns minus a term that adjusts for the impact of growth. While this result is not fundamentally new, it has been ignored by most studies that have employed stationary economies.

Ignoring economic growth can have substantial consequences for asset pricing models. Assume that the mean annual log real return to equity equals 6.5% and that the economic growth rate equals 3%. Adjusting for growth implies that a stationary model with a mean log return to equity of 6.5% corresponds to an economy with a mean log return of 9.46% (=6.5% + log(1.03)), substantially greater than the observed rate of return in the data. Thus, the return to equity in the stationary model economies should be the desired value minus the correction for growth. This indicates that the stationary models in the literature that match the observed return to equity actually correspond to real economies with much higher returns.

This argument also applies to the risk-free rate. The production-based asset pricing models such as Gomes and Schmid (2008) and Livdan, Sapriza, and Zhang (2008) will have more difficulty generating a low risk-free rate once they incorporate economic growth. This arises due to the fact that the pricing kernel times $(1+\gamma)$ enters into the contraction mapping of the Bellman equation. Thus, convergence of the value function requires that the average of the above term be below one, placing an implicit upper bound of the risk-free rate close to the growth rate. The degree of this difficulty depends on whether the target risk-free rate equals the long-term average of 1.8% reported in Campbell, Lo, and MacKinlay (1997) or the lower risk-free rates observed in more recent data.

The analysis does not, however, imply that excess returns, such as the equity premium or value premium, require an adjustment for for economic growth. This arises due to the fact that the growth adjustment is necessary for all asset returns and, as such, differences in asset returns remain unchanged when stationary economies are mapped to economies with growth.

3 Data

The data for the estimation is obtained from the Compustat Fundamentals data set. I estimate the model on both an annual data set and a quarterly data set. The sample periods for the annual and quarterly data set extend from 1967 to 2007 and 1984/1/1 to 2008/06/31, respectively. The start dates are constrained by the lack of data on shareholders equity and dividends prior to the start dates for the annual and quarterly data sets, respectively. The sample excludes financial firms and regulated utilities as the model would not be appropriate to use for such firms. The above model characterizes the behavior of a single representative firm. Using an aggregate firm has the benefit that cross-sectional heterogeneity has little impact on the key variable of interest: the expected return to equity. The drawback from working with a representative firm is that the other structural parameters may not necessarily describe the behavior of a single firm taken in isolation. The data sets for the representative firm are constructed by aggregating firm level data on total assets, capital expenditures, common dividends, income accruing to shareholders, total liabilities, corporate taxes and the book and market values of equity from the samples. Any firm with missing values for the market value of equity is excluded from the aggregation.

Panel A of Table 1 reports the summary statistics of interest for the representative firm at the quarterly and annual frequencies. All variables except the market-to-book ratio, leverage and interest expenses are constructed after scaling by lagged total assets. The market-to-book ratio equals the market value of common equity dividend by book equity. The mean value of earnings accruing to shareholders indicates that the aggregate firm earns close to 1% on its total assets each quarter. However, earnings are volatile over the business cycle with negative aggregate earnings in a few quarters. Mean dividends are significantly smaller and less volatile than earnings, suggesting that firms fund some investment through retained earnings and smooth dividends. Dividends are less smooth relative to earnings at the annual frequency. Almost 2/3rds of the total assets of the aggregate firm are financed through borrowing, indicating the importance of leverage in understanding equity returns (see Gomes and Schmid (2008)).

3.1 Calibrated parameters

The above model includes many auxiliary parameters, such as the corporate tax rate. One approach would be to include relevant information and estimate all of these parameters. A simpler approach would be to calibrate some of the model parameters so as to match the data, and estimate the rest. This has the benefit of focusing the estimation on the parameters of interest, and improves their identification.

Panel B of Table 1 reports the calibrated values for the auxiliary parameters at the quarterly and annual frequencies. The depreciation rate is set equal to the ratio of mean aggregate depreciation to total assets over time. This is lower than the depreciation rates obtained as a ratio of capital stocks, as the total assets of firms are more than twice the total physical capital stock. The interest rate equals the ratio of mean interest payments to total liabilities. The linear tax rate is equal to mean aggregate taxes to taxable income. The calibrated tax rate is close to the federal tax rate of 35% plus the average state tax rate of 4%. There is no optimal leverage in the model as firms do not face a trade-off in selecting their debt level.[10] Therefore, I calibrate total borrowing using the mean of the book value of leverage. Finally, the fixed cost of operations equals the ratio of selling, general, and administration expenses to total assets.

This study also highlights the importance of incorporating growth in structural models. As such, I report results from the estimation assuming quarterly growth rates of 0.5% and 0.75%, respectively. The growth rate of 0.75% corresponds to the mean real GDP growth rate over the sample period. The lower growth rate of 0.5% reflects per-capita real GDP growth over the same period. The annual estimates assume a growth rate of 3%.

[10] Allowing for the interest cost to vary with the profitability level does not result in an optimal leverage choice in the model as the interest cost and the tax benefit both have linear effects on firm value.

3.2 Identification of model parameters

The simulated method of moments estimation involves matching moments from the data to those obtained by simulating the model given in equation (5). The matched moments include the mean, variance, and autocorrelation of income and dividends plus the mean of the market-to-book ratio and the variance of investment.[11] Given the black box nature of the estimation, it is perhaps worth providing some intuition on which moments help identify which parameters.

The average income level helps pin down the curvature of the profit function, θ. This reflects the link between θ and the markup charged by the firm. The autocorrelation of dividends and income helps pin down the autocorrelation parameter, ρ. Similarly, the variance of income and the variance of dividends both include the volatility of the productivity shock process, σ. This leads to some tension in the estimation as it cannot simultaneous match both income and dividend variances. The mean dividend level helps pin down the adjustment cost parameter, λ. The resource constraint for the firm implies that dividends equals income minus the cost of investment. Given an level of earnings and investment, an increase in adjustment costs will lower the dividend payout.

Once the model has pinned down the above parameters, the average market-to-book ratio helps pin down the mean pricing kernel. Effectively, given income and dividends, a lower market-to-book ratio generates a lower mean pricing kernel. The variation within the pricing kernel influences the volatility of investment as the first order condition relates investment to expected firm values.

Although the above discussion focused on one-to-one mappings between the moments and the parameters, the estimation employs data on all the moments to pin down all the parameters. In the model simulations, a parameter change will directly or indirectly affect all the moments. For example, an increase in ρ will lead to high average market-to-book ratios and higher investment volatility, in addition to its direct effect on the autocorrelation moments.

4 Results

This section presents the results from estimating the model given in (5). The first subsections estimate the model using quarterly data. The next subsections estimate the model using annual data and include results obtained after splitting the annual sample in 1987.

[11] The mean of investment in the simulations equals the depreciation rate plus the growth rate.

4.1 Constant pricing kernel

Classic valuation studies (Samuelson (1965), Samuelson (1973) and Shiller (1981)) have argued for assuming a constant pricing kernel. Further, time variation in equity premium may not have much impact on estimating mean expected returns. Thus, I first present results obtained by assuming that $M_{t,t+1} = \beta$, and subsequently examine the impact of allowing the pricing kernel to vary with x_t.

Panel A of Table 2 presents the parameter estimates from this estimation. The expected mean return to equity from the estimation equals 4.9%, lower than the historical average real return of 6.5% used in the literature.[12] On the other hand, the estimated value is higher the realized mean return for the twenty five year period from 1969 to 2003 reported by Cogley and Sargent (2008). The estimates indicate that the expected equity premium is lower than historical values, consistent with the findings of Fama and French (2002), Siegel (1999), Claus and Thomas (2001) and Jagannathan, McGrattan, and Scherbina (2001). These authors argue that historical realized returns are likely to be higher than expected returns in the future. This could be due to either decreasing macroeconomic risk (see Lettau, Ludvigson, and Wachter (2008)), decreased risk aversion (see Cogley and Sargent (2008)) or changes in taxes (see McGrattan and Prescott (2005)).

One caveat that should be noted is that these results are predicated on the above, albeit standard, model for the aggregate firm. Using a different model may yield different estimates for the expected real return to equity. However, the results obtained using this method are quite similar to the survey-based evidence on expected returns reported by Graham and Harvey (2005).

The intuition for the estimated expected real return to equity is that the data on earnings, dividends, and associated growth options can only be reconciled with the data on valuations with a low discount rate. A higher expected return would lower the discounted present value of future dividend streams, resulting in a smaller aggregate market-to-book ratio. The precision of the estimate arises from the sensitivity of the model to the expected return. Thus, the method employed in the study helps filter out some of the noise in observed returns.

The other parameter estimates would not necessarily be comparable to those obtained from estimating firm level models due to aggregation effects. The point estimate for θ indicates some role for capital augmenting technological change or some flexibility in the aggregate supply of labor. The estimates for ρ and σ reflect the data on the income process, while the estimate for λ enables

[12] Eberly, Rebelo, and Vincent (2008) estimates a discount rate of 5% for a sample of large US corporations. Their estimates do not take into account the effect of leverage.

the model to match the data on dividends. Note that the investment adjustment costs are the only frictions in the model, and the estimates reflect the gap between income, dividends and investment observed in the data.

Panel B of Table 2 presents the moments used in the estimation. The model moments are constructed using the parameter values reported in Panel A. The model successfully matches the first moments of income, dividend and the market-to-book ratio in the data. However, the model fails to generate the observed variability in each of the series. This arises partly as a result of all variation in the model being driven by a single shock process. The failure of the model to match the variances also drives up the goodness-of-fit statistic $\hat{\Psi}$.

4.2 Time varying pricing kernel

This subsection presents the results obtained from estimating the model after relaxing the assumption of a constant pricing kernel. I parameterize the log pricing kernel as a log-linear function of current and future aggregate conditions:

$$\log(M_{t,t+1}) = -(1+b_0)\log(1+\gamma) - b_1(\log(z_t) - \log(z_{t+1})) - b_2(\log(z_t) - \mu_z),$$

where μ_z denotes the mean of $\log(z)$. The above parametrization is similar to the one employed by Berk, Green, and Naik (1999). b_1 captures the impact of changes in aggregate conditions on the pricing kernel while b_2 captures time variation in mean expected returns with the aggregate state. The pricing kernel relevant for the model in the detrended economy equals the above term times $(1+\gamma)$ in order to correctly account for growth in the valuation Eq. (5). I scale the constant term, b_0, by the growth rate for convenience.

Panels A and B of Table 3 reports the estimated parameter values and the moments employed in the estimation, respectively.[13] The estimates imply a mean annual expected real return to equity of 5.1%, similar to the estimate obtained with the constant pricing kernel. This indicates that time variation in the pricing kernel, by itself, does not impact the estimated mean return to equity. The intuition behind this result is that the expected mean return is pinned down primarily by the first moments of earnings, dividends and market-to-book values. Incorporating time variation in expected returns does, however, improve the fit of the model with regards to the variability of

[13]These estimates were obtained with randomly chosen starting values, independent of the estimates of the previous section. The similarity between the estimates gives confidence that the method converges to the true parameter values.

those variables. As before, the result indicates that, expected real returns to equity over the sample period were lower than their historical values.

The other parameter values are mostly similar to those obtained with a constant $M_{t,t+1}$. The estimated value for λ remains higher than the counterparts from firm level estimations, partly due to its role as the sole friction in the model. The mean expected return estimate has a small standard error, even though not all components of the pricing kernel are precisely estimated. This arises due to the fact that the pricing kernel is a non-linear transformation of $b_0, b_1, \text{and} b_2$. The remainder of this study employs the model with the time-varying pricing kernel.[14]

4.3 A lower growth rate

The previous results assume a quarterly economic growth rate of 0.75%. However, one can argue that the appropriate growth rate in the model corresponds to the per capital growth rate of the economy. This subsection reports the results obtained by estimating the model with a quarterly growth rate of 0.50%, equal to the per capital GDP growth rate over the sample period. The pricing kernel varies with the aggregate conditions and has the same parametrization as above.

Panels A and B of Table 4 reports estimated parameter values and the moments employed in the estimation, respectively, assuming a growth rate of $\gamma = 0.5\%$. The estimates for the pricing kernel parameters imply a mean annual expected real return to equity of 3.9%, lower than the estimated obtained with the higher growth rate. As before, the expected real return to equity estimated over the sample period is lower than the mean historical return. The intuition for the decline in the expected return is that, *ceteris paribus*, a lower growth rate reduces the value of the aggregate firm. A decline in expected returns counteracts this by increasing the value of future dividend payments. The difference in the estimates highlight the necessity of incorporating economic growth into the framework.

The estimates for the other parameters remain similar to those obtained above, except for the adjustment cost parameter. The increase in λ arises from the reduction in the steady state investment level, which equals $\delta + \gamma$, as a result of the lower growth rate. As the estimation algorithm aims to fit both dividends and income data, the investment friction increases in order to offset the lowered investment levels. This suggests that the model with $\gamma = 0.75$ fits the data better from an economic perspective.

[14] See Wachter and Warusawitharana (2009) for evidence on time variation in the equity premium.

4.4 Annual data

The previous analysis suggested that expected returns over the past twenty five years were lower than realized returns over long time periods. The reader may wonder if this reflects a failure of the model to generate high expected returns. Estimating the model using a annual data over a longer time period helps tackle this concern. Using annual data also enables an analysis of whether expected returns declined over the sample period.

Panels A and B of Table 5 present the estimated parameter values and the matched moments from the estimation, respectively, from the estimation using annual data. The assumed economic growth rate equals 3%. Limitations on the book value of shareholder's equity data restrict the sample start date to 1967. The estimated expected real return to equity is higher than the value obtained from the quarterly sample. It is also closer to the annual return of 6.5% to 7% that models typically match. The results suggest that the expected real return to equity has declined from its historical levels.

The estimates for the other parameters are driven by the matched moments and generally comparable to the quarterly estimates. The lower estimate for the adjustment cost parameter, *lambda*, suggests that investment frictions are less binding at an annual horizon compared to a quarterly horizon. As before, the mean expected return is fairly precisely estimated, though the model has difficulty pinning down some of the pricing kernel parameters. The model fares better at matching the volatilities of dividends, income and investment with the annual data as annual dividends appear to be less smooth than quarterly dividend payments.

4.5 Split sample

Estimating the model after separating the sample into two periods provides a further test of the hypothesis that expected real returns to equity has declined. Table 6 presents the results from estimating the model after splitting the sample at its midpoint, 1987. Panel A reports the parameter estimates from the two samples, and Panel B reports the matched moments.

The split sample results provide clear evidence of a decline in the expected real return to equity, which was 8% in the twenty years from 1967-86, before declining to 5.4% for the sample from 1987 onwards. The estimated decline in the expected return to equity is economically significant. The findings suggest that either expected returns to equity change slowly over long horizons or that there was a structural break in expected returns across the two sample periods.

The estimates for the model parameters are comparable to those obtained with the full sample. In general, the smaller sample size results in higher standard errors with some of the pricing kernel parameters estimates having a large standard error. This carries over somewhat to the precision of the expected return, which is nonetheless small due to the sensitivity of the model to changes in mean expected returns. The model manages to fit the volatilities of dividends, income and investment fairly well over both sample periods.

4.6 Robustness

The previous sections demonstrated that the estimated expected return to equity over the past twenty five years varies from 4% to 5%, depending on the assumed economic growth rate. Additional robustness checks on allowing a non-linear tax rule, assuming leverage varies with lagged profitability and incorporating a time-varying interest rate lead to similar results on the expected return to equity.

One area in which the model fails to match the data is the variance of the market-to-book ratio. Including this as an additional moment condition has little impact on the results, as the estimation mostly ignores this moment. This reflects the difficulty of matching variances of different orders of magnitude using a single shock process. A more economic interpretation of this would be that the model fails to generate the volatility in the market-to-book ratio arising from the sharp rise and subsequent drop of stock prices during the tech boom years.

Overall, the estimation pins down the expected return to equity quite precisely in the range of 4% to 5%. These findings support other evidence indicating that expected returns to equity have declined and suggest that investors should not necessarily expect future returns from equity to necessarily be similar to historical returns.

5 Model implications

This section evaluates the model along some dimensions not used in the estimation and examines the implications of the findings for the portfolio choice decisions.

5.1 Model evaluation

The model presented in this study is primarily aimed towards estimating expected returns to equity. The reader may be interested in understanding how the model fares with regards to features of asset prices that were not employed in the estimation.[15] Such a comparison serves as an out-of-sample evaluation of the model. It also helps highlight some strengths and weaknesses of the model.

Table 7 presents some statistics of interest obtained from the data and the model simulations. The parameter estimates underlying the model simulations are those reported in Section 4.2. The data statistics are obtained from the data set used in that estimation. The statistics relate to the key variables employed in the estimation: aggregate dividends, income, and market value.

The model generates an annualized equity return volatility of 12%, fairly close to the value observed in the data. This indicates that although the model has trouble matching all the volatilities in the data, it has some success generating a high equity return volatility.

The valuation ratios derived from the model correspond to those of the aggregate equity market. The asset pricing literature typically focuses on valuation ratio per share outstanding. As Bansal and Yaron (2007) demonstrate, these valuation ratios may differ when the number of shares in the economy varies over time. They argue that the log aggregate market value to aggregate dividends series provides helpful information that is not captured by the per share price-dividend ratio.

The log market value to dividend ratio from the model is less volatile than the corresponding data counterpart. This reflects the model's inability to generate sufficient volatility in the market value of equity. On the other hand, the model comes close to capturing the autocorrelation of the log market value to dividend ratio at a quarterly frequency. As Bansal and Yaron (2007) find, the log value to dividend ratio exhibits less persistence than the per share price-dividend ratio.

The quarterly dividend growth volatility from the model closely matches the data value. However, the dividend growth series is more negatively autocorrelated in the data than the model. This suggests that dividend growth rates face transitory shocks in the data that the model does not fully capture. The model generates a high contemporaneous correlation between dividends and income, reflecting the role of income as a key state variable in the model. The corresponding data value is much lower, perhaps indicating that firms smooth dividends in response to quarterly income shocks. On the other hand, the correlation between dividends and investment in the model closely matches the corresponding data value.

One dimension along which the model clearly fails to match the data is the risk-free rate.

[15] I thank João Gomes for this suggestion.

As discussed in Section 2.2, production-based models with time-separable utility functions cannot generate low risk-free rates once one accounts for economic growth. However, this limitation does not affect the ability of the model to estimate the expected return to equity.

5.2 Implications for portfolio choice

This section examines the portfolio choice implications of the estimated return to equity. Holding all else constant, it examines the impact of the estimated expected return to equity on the optimal exposure to the stock market compared to that obtained by using a historical average for equity returns.[16]

Figure 1 plots the optimal allocation to equity for a CRRA investment as a function of his risk aversion coefficient. The portfolio weight using historical data was constructed assuming an annualized real return to equity of 6.5% and a risk-free return of 0.5%. The excess return of 6% reflected the post-war excess return to equity. The expected volatility of the stock return was set equal to its sample average of 16%. The dashed line represents the optimal equity holding for an investor using the estimated real return to equity of 5.1%. This investor uses the same values for the risk-free rate and the volatility of equity returns as the data-based investor.

Portfolio allocations based on the estimated expected return generates a lower exposure the stock markets. The gap between the two portfolios is particularly pronounced for low levels of risk-aversion. An investor with a relative risk aversion coefficient of 4 would hold 55% of his wealth in equities given an expected return of 5.1%, compared to nearly 70% of his wealth in equities for an expected return of 6.5%. Increased risk aversion reduces the gap as the investor becomes less willing to hold equities. The analysis demonstrates that the finding of a lower expected real return to equity has sharp implications for the optimal portfolio choice of an investor.

6 Conclusion

This study presents estimates of the expected real return to equity using a production-based asset pricing approach. The intuition underlying the approach is that the expected returns provide a mapping from data on earnings, dividends onto the valuation of firms. Given data on these variables

[16]This analysis does not compare the portfolio choice of an investor who lives within the model to that implied by the data. Such a comparison would take into account the model estimated equity return as well as the implied risk-free rate and volatilities. The above comparison focuses more sharply on the effect of the difference between estimated expected returns and historical average values.

and a dynamic model for their evolution over time, the study estimates the expected return that would generate the best fit for the model.

Using this approach, this study estimates an expected real return to equity in the range of 3.9% to 5.1% over the past twenty five years, depending on the assumed economic growth rate. These results are substantially lower than mean historical returns to equity, suggesting that investors expect lower future returns to equity than have been realized in the past. The findings are consistent with other studies in the literature that have argued that the equity premium has declined over time.

The findings may also be of practical relevance in that investors form their portfolio holdings based on expected real returns to equity. The estimated expected returns to equity imply a substantially smaller portfolio allocation to equities than would be obtained using historical sample averages. A lower expected return to equity also translates to lower expected portfolio returns, indicating that investors may need to increase their savings rate in order to finance retirement expenses in a life-cycle framework.

One interesting question that this study cannot answer is whether the decline in expected returns reflects a permanent change or a temporary regime shift as in Lettau, Ludvigson, and Wachter (2008). It is quite plausible that, in light of the recent financial crisis, expected returns to equity have increased again as investors revise upward their views on the riskiness of equities. The method in this study can not evaluate this question without data on a sufficient sample period following the crisis.

The theoretical analysis also highlights the need for incorporating economic growth into stationary asset pricing models. The implied returns to equity in these models should equal the desired return minus the growth rate. The current literature fails to include this correction. As such, a stationary model with a 6.5% return to equity will imply a real return to equity of close to 10% when mapped onto observed data, given a 3% growth rate. Incorporating a growth term when calibrating stationary production-based models avoids a substantial error.

Appendix

A The representative firm model

The output produced by the representative firm is given by a Cobb-Douglas specification with capital share equal to α:

$$Y(K_t, L_t, X_t, Z_t) = Z_t K_t^{\alpha} (X_t L_t)^{1-\alpha}.$$

Without loss of generality, the technological progress is assumed to be labor augmenting and to grow at a constant rate γ.

$$X_{t+1} = X_t(1+\gamma).$$

The firm faces a downward sloping demand curve for its output, with a constant elasticity of substitution ν. The price per unit of output is given by

$$P_t = d_t Y(K_t, L_t, X_t, Z_t)^{-\nu},$$

where d_t measures the current level of demand for the good. The per period profits of the firm equals revenue minus labor costs minus a fixed cost of operations,

$$\Pi(K_t, L_t, X_t, Z_t) = \max_{L_t} P_t Y(K_t, L_t, X_t, Z_t) - W_t L_t - cX_t.$$

The real wage rate, W_t, adjusts such that the demand for labor, L_t, equals the fixed supply, L.

A balanced growth path is one in which capital, output, profits and wages grow at constant, possibly different, rates. A balanced growth is necessary in order to translate this model into a stationary economy. However, this imposes an additional requirement:

Proposition 2 *A balanced growth in the steady state with a constant growth rate in all variables requires that*

$$\frac{d_{t+1}}{d_t} = (1+\gamma)^{\nu}.$$

Proof. The profits of the firm are obtained as the solution to the following optimizing problem

$$\Pi(K_t, L_t, X_t, Z_t) = \max_{L_t} d_t Y(K_t, L_t, X_t, Z_t)^{1-\nu} - W_t L_t - cX_t,$$

subject to the market clear condition for labor. The first order condition yield

$$d_t(1-\nu)Y(K_t, L_t, X_t, Z_t)^{-\nu}\frac{\partial Y(K_t, L_t, X_t, Z_t)}{\partial L_t} - W_t = 0.$$

Using the Cobb-Douglas specification to simplify the derivative and rearranging terms, one obtains

$$d_t(1-\nu)(1-\alpha)Y(K_t, L_t, X_t, Z_t)^{1-\nu} = W_t L_t.$$

The profits of the firm can therefore be written as:

$$\Pi(K_t, L_t, X_t, Z_t) = d_t Y(K_t, L_t, X_t, Z_t)^{1-\nu}\left[1-(1-\nu)(1-\alpha)\right] - cX_t. \tag{A.1}$$

In order for $\Pi(K_t, L_t, X_t, Z_t)$ to exhibit a constant steady state growth, $d_t Y(K_t, L_t, X_t, Z_t)^{1-\nu}$ must grow at the same rate as X_t. A constant growth rate implies that

$$\frac{d_{t+1}}{d_t} = \gamma^{\nu}.$$

■

The growth in the demand shift parameter ensures that wages and sales grow at a constant rate.

Corollary 1 *The profit function after substituting the optimal labor choice is given by:*

$$\tilde{\Pi}(K_t, X_t, z_t) = z_t K_t^{\theta} X_t^{1-\theta} - cX_t. \tag{A.2}$$

Proof. As the demand shocks growth proportionally with γ, we can write

$$d_t = c_0 X_t^{\nu}.$$

Substitute the expression for $Y(K_t, L_t, X_t, Z_t)$ into the profit function given in (A.1) and gather terms into the trend deviation term z_t to yield

$$\begin{aligned}\tilde{\Pi}(K_t, X_t, z_t) &= z_t K_t^{\theta} X_t^{1-\theta} - cX_t, \\ \theta &= \alpha(1-\nu).\end{aligned}$$

■

B Asset returns in models with and without growth

This subsection presents an alternate proof of Proposition 1 using only the representative consumer's budget constraint, as opposed to the firm value based proof presented in the main text.

Proof. Denote the utility function of the representative consumer by $U(C_t)$, where C_t denotes aggregate consumption. The per period budget constraint of the representative agent is given by

$$C_t + (V_t - D_t)S_t = V_t S_{t-1} + W_t L, \tag{A.3}$$

where S_t denotes the holding of the firm. Given a rate of time preference parameter b, the representative agent aims to maximize the following:

$$\max E\left[\sum_{t=0}^{t=\infty} b^t U(C_t)\right]. \tag{A.4}$$

The first order conditions for asset returns in the economy with growth is given by

$$1 = E\left[b\frac{U_c(C_{t+1})}{U_c(C_t)} R_{t,t+1}\right]. \tag{A.5}$$

Now consider the budget constraints in the detrended economy without growth.

$$c_t + (v_t - d_t)S_t = v_t S_{t-1} + w_t L, \tag{A.6}$$

where the asset holding, S_t, and labor supply are not affected by the detrending. The detrended consumption level is given by:

$$c_t = \frac{C_t}{X_t}.$$

The objective function of the agent in terms of the variables in the detrended economy is:

$$\max E\left[\sum_{t=0}^{t=\infty} b^t U(c_t X_t)\right]. \tag{A.7}$$

Note that one can not simply replace the consumption term in Eq. (A.4) with the detrended term to obtain (A.7). This arises from the fact that the anticipated growth in consumption through technological progress impacts the agent's utility. The first order conditions using the detrended term imply that

$$1 = E\left[b\frac{U_c(c_{t+1}X_{t+1})X_{t+1}}{U_c(c_t X_t)X_t}\tilde{R}_{t,t+1}\right]. \tag{A.8}$$

Given that $C_t = c_t X_t$ and that the first order conditions given in Eqs. (A.5) and (A.8) must both hold, one obtains that

$$\begin{aligned} R_{t,t+1} &= (1+\gamma)\tilde{R}_{t,t+1} \\ \Rightarrow \qquad \log R_{t,t+1} &= \log \tilde{R}_{t,t+1} + \log(1+\gamma). \end{aligned}$$

■

C Simulated method of moments

The simulated method of moments estimator of Lee and Ingram (1991) and Duffie and Singleton (1993) obtains parameter estimates by matching a set of selected moments from the data to those obtained by simulation. Denote the true values of the structural parameters by Ψ^*. The matched moments can be written as a solution to a minimization problem $Q(Y, M)$, where Y denotes the data and M the moments to be matched. The data moments are then given by

$$\hat{M} = \arg\min_M Q(Y_N, M), \tag{A.9}$$

where Y_N denotes a data matrix with N observations. The corresponding moments for the simulated data set with parameter vector Ψ and $n = N \times S$ observations are given by

$$\hat{m}(\Psi) = \arg\min_M Q(Y_n, M). \tag{A.10}$$

This study picks $S = 100$, which is above the recommended minimum.

The structural parameters are then obtained by minimizing a quadratic form of the distance between the data and simulated moments.

$$\hat{\Psi} = \arg\min_{\Psi} N \left[\hat{M} - \hat{m}(\Psi)\right]' \hat{W} \left[\hat{M} - \hat{m}(\Psi)\right], \tag{A.11}$$

where $\hat{W}$ denotes a positive definite weighting matrix. The value of the above function at the minimum, denoted by $\hat{\Phi}$, provides a goodness-of-fit measure. The optimal weighting matrix is given by

$$\hat{W} = \left[N \text{var}(\hat{M})\right]^{-1}. \tag{A.12}$$

The above covariance matrix is calculated with the actual data set using the influence function method of Erickson and Whited (2000). The estimator is asymptotically normal for fixed S with covariance matrix given by

$$\begin{aligned} \sqrt{N}(\hat{\Psi} - \Psi^*) &\sim \mathsf{N}(0, \Sigma) \\ \Sigma &= (1 + \frac{1}{S}) \left[\frac{\partial^2 Q}{\partial \Psi \partial M'} \left(\frac{\partial Q}{\partial M} \frac{\partial Q}{\partial M}'\right)^{-1} \frac{\partial^2 Q}{\partial M \partial \Psi'}\right]^{-1}. \end{aligned} \tag{A.13}$$

While $\frac{\partial Q}{\partial M}$ can be evaluated analytically, numerical methods are required to obtain $\frac{\partial^2 Q}{\partial \Psi \partial M}$. Both partial derivatives are computed using simulated data evaluated at the data moments.

D Numerical solution

The simulations require a numerical solution of the value function for the aggregate firm. The capital grid has 150 points and the profitability grid has 10 points. The capital grid is centered around an approximation of the median size of the firm given the parameters. The approximate value of the steady state capital stock ($\hat{k}$) for the case of the constant pricing kernel is given by the following:

$$\begin{aligned} \hat{k} &= \left(\frac{\theta * \mu_z * (1 - \tau)}{\text{usr}}\right)^{1/(1-\theta)}, \\ \mu_z &= \exp\left(\mu/(1-\rho) + 0.5 * \sigma^2/(1-\rho^2)\right), \\ \text{usr} &= 1/\beta - 1 + \delta - \tau * \delta + \lambda * (\delta + \gamma) * (1/\beta - 1 + \delta - .5 * (\delta + \gamma), \end{aligned}$$

where μ_z and usr denote the mean profitability level and the steady state Jorgensonian user cost of capital, respectively.[17] A check reveals that the steady state firm size obtained from the simulations lies very close to this approximation. The profit grid is formed using the quadrature method of Tauchen and Hussey (1991).

The simulated sample is generated using the value and policy functions for the aggregate firm. The law of motion for profitability is generated directly using the transition equations (2). The simulation is run for 20,000 years, with the initial 10,000 discarded as a burn-in sample. The value of the quadratic form of the distance between the data moments and simulated moments is computed for each simulation. The program searches for the parameters that minimize this distance using the simulated annealing algorithm. Each estimation involved evaluating more than 10,000 candidate parameter sets and took about a week of computing time.

[17]The details of this calculation are available from the author.

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Table 1: Summary statistics

Panel A reports the summary statistics for the aggregate firm constructed by aggregating the firm level data from Compustat at a quarterly frequency. 'St. dev.' denotes standard deviation. The sample periods for the quarterly and annual data are from 1984/1/1 to 2008/06/30 and 1967/1/1 to 2007/12/31, respectively. The earnings variable measures the income accruing to common shareholders scaled by lagged total assets. Market-to-book equals the aggregate market value of equity scaled by the book value of equity. Leverage equals total liabilities divided by total assets. Income, dividends, investment, interest costs, corporate taxes and depreciation are all scaled by lagged total assets and reported as percentages. Panel B reports the parameters that are calibrated to match the corresponding sample data means in the estimations with quarterly and annual data sets.

Panel A: Sample moments

	Quarterly data		Annual data	
Variable	Mean	St. dev.	Mean	St. dev.
Earnings	0.94	0.45	4.99	1.76
Market-to-book	2.95	0.91	2.30	1.06
Dividends	0.47	0.11	2.29	0.57
Investment	1.71	0.39	8.63	2.28
Leverage	0.65	0.03	0.60	0.07
Interest costs	0.89	0.22	4.17	0.95
Corporate taxes	0.69	0.17	3.95	1.55
Depreciation	1.15	0.12	5.18	0.51

Panel B: Calibrated parameter values

Parameter	Quarterly data	Annual data
Depreciation rate (δ)	1.15%	5.18%
Interest rate (r)	0.89%	4.17%
Corporate tax rate (τ)	40.2%	40.2%
Book leverage	0.65	0.600
Fixed costs to assets	3.93%	17.53%
Growth rate (γ)	{0.5%, 0.75%}	3.00%

Table 2: Constant pricing kernel

Panel A reports the parameters values obtained from estimating the model on the aggregate firm under the assumption of a constant pricing kernel. Panel B reports the corresponding moment values from the data and the model. The data moments are obtained using data on the aggregate firm constructed by summing the variables across all firms in a given quarter. Income, dividends and investment are all reported as percentages. The sample period is from 1984/1/1 to 2008/06/30. $\hat{\Psi}$ denotes a goodness-of-fit measure. The quarterly economic growth rate, γ, is assumed to be 0.75%. The pricing kernel is parameterized as

$$M_{t,t+1} = \beta.$$

The applicable pricing kernel for the detrended model equals $M_{t,t+1}$ times $(1+\gamma)$, in order to correctly adjust for the impact of growth. The standard error for the expected real annual return to equity, $E[r]$, is computed using the delta method.

Panel A: Parameters

Parameter	θ	ρ	σ	λ	β	$E[r]$	$\hat{\Psi}$
Estimate	0.44	0.74	0.015	14.10	0.9905	4.90	74.41
Standard error	(0.06)	(0.05)	(0.001)	(1.37)	(0.0019)	(0.08)	

Panel B: Moments

Moment	Data	Model
Mean of income	0.94	1.01
Variance of income	0.10	0.01
Mean of market-to-book	2.95	3.07
Mean of dividends	0.47	0.49
Variance of dividends	0.01	0.01
Variance of investment	0.15	0.00
Autocorrelation of profitability	0.88	0.75
Autocorrelation of dividends	0.77	0.75

Table 3: Time-varying pricing kernel

Panel A reports the parameters values obtained from estimating the model on the aggregate firm under the assumption of a time varying pricing kernel. Panel B reports the corresponding moment values from the data and the model. The data moments are obtained using data on the aggregate firm constructed by summing the variables across all firms in a given quarter. Income, dividends and investment are all reported as percentages. The sample period is from 1984/1/1 to 2008/06/30. $\hat{\Psi}$ denotes a goodness-of-fit measure. The quarterly economic growth rate, γ, is assumed to be 0.75%. The pricing kernel is parameterized as

$$\log(M_{t,t+1}) = -(1+b_0)\log(1+\gamma) - b_1(\log(z_t) - \log(z_{t+1})) - b_2(\log(z_t) - \mu_z),$$

where μ_z denotes the mean of $\log(z)$. The applicable pricing kernel for the detrended model equals $M_{t,t+1}$ times $(1+\gamma)$, in order to correctly adjust for the impact of growth. The standard error for the expected real annual return to equity, $E[r]$, is computed using the delta method.

Panel A: Parameters

Parameter	θ	ρ	σ	λ	b_0	b_1	b_2	$E[r]$	$\hat{\Psi}$
Estimate	0.44	0.79	0.028	10.62	0.29	-0.06	-0.10	5.11	46.46
Standard error	(0.01)	(0.05)	(0.004)	(1.33)	(0.05)	(0.46)	(0.12)	(0.12)	

Panel B: Moments

Moment	Data	Model
Mean of income	0.94	0.94
Variance of income	0.10	0.04
Mean of market-to-book	2.95	2.98
Mean of dividends	0.47	0.49
Variance of dividends	0.01	0.01
Variance of investment	0.15	0.04
Autocorrelation of profitability	0.88	0.79
Autocorrelation of dividends	0.77	0.89

Table 4: Lower economic growth rate

Panel A reports the parameters values obtained from estimating the model on the aggregate firm under the assumption of a time varying pricing kernel. Panel B reports the corresponding moment values from the data and the model. The data moments are obtained using data on the aggregate firm constructed by summing the variables across all firms in a given quarter. Income, dividends and investment are all reported as percentages. The sample period is from 1984/1/1 to 2008/06/30. $\hat{\Psi}$ denotes a goodness-of-fit measure. The quarterly economic growth rate, γ, is assumed to be 0.50%. The pricing kernel is parameterized as

$$\log(M_{t,t+1}) = -(1+b_0)\log(1+\gamma) - b_1(\log(z_t) - \log(z_{t+1})) - b_2(\log(z_t) - \mu_z),$$

where μ_z denotes the mean of $\log(z)$. The applicable pricing kernel for the detrended model equals $M_{t,t+1}$ times $(1+\gamma)$, in order to correctly adjust for the impact of growth. The standard error for the expected real annual return to equity, $E[r]$, is computed using the delta method.

Panel A: Parameters

Parameter	θ	ρ	σ	λ	b_0	b_1	b_2	$E[r]$	$\hat{\Psi}$
Estimate	0.43	0.81	0.025	21.08	0.58	-0.39	-0.03	3.94	62.95
Standard error	(0.02)	(0.07)	(0.007)	(2.04)	(0.11)	(4.95)	(1.01)	(0.16)	

Panel B: Moments

Moment	Data	Model
Mean of income	0.94	0.94
Variance of income	0.10	0.03
Mean of market-to-book	2.95	3.12
Mean of dividends	0.47	0.48
Variance of dividends	0.01	0.01
Variance of investment	0.15	0.02
Autocorrelation of profitability	0.88	0.81
Autocorrelation of dividends	0.77	0.86

Table 5: Time-varying pricing kernel (Annual data)

Panel A reports the parameters values obtained from estimating the model on the aggregate firm under the assumption of a time varying pricing kernel using annual data. Panel B reports the corresponding moment values from the data and the model. The data moments are obtained using data on the aggregate firm constructed by summing the variables across all firms in a given year. Income, dividends and investment are all reported as percentages. The sample period is from 1967/1/1 to 2007/12/31. $\hat{\Psi}$ denotes a goodness-of-fit measure. The annual economic growth rate, γ, is assumed to be 3%. The pricing kernel is parameterized as

$$\log(M_{t,t+1}) = -(1+b_0)\log(1+\gamma) - b_1(\log(z_t) - \log(z_{t+1})) - b_2(\log(z_t) - \mu_z),$$

where μ_z denotes the mean of $\log(z)$. The applicable pricing kernel for the detrended model equals $M_{t,t+1}$ times $(1+\gamma)$, in order to correctly adjust for the impact of growth. The standard error for the expected real annual return to equity, $E[r]$, is computed using the delta method.

Panel A: Parameters

Parameter	θ	ρ	σ	λ	b_0	b_1	b_2	$E[r]$	$\hat{\Psi}$
Estimate	0.46	0.71	0.037	4.00	0.46	-0.51	-0.20	6.06	14.39
Standard error	(0.03)	(0.03)	(0.005)	(0.43)	(0.23)	(5.26)	(1.62)	(0.45)	

Panel B: Moments

Moment	Data	Model
Mean of income	4.99	4.91
Variance of income	1.88	1.11
Mean of market-to-book	2.30	2.55
Mean of dividends	2.29	2.31
Variance of dividends	0.31	0.33
Variance of investment	5.06	2.49
Autocorrelation of profitability	0.75	0.71
Autocorrelation of dividends	0.86	0.90

Table 6: Split sample

Panel A reports the parameters values obtained from estimating the model on the aggregate firm for two annual sub-samples. The pricing kernel varies over with the current and next period aggregate state. Panel B reports the corresponding moment values from the data and the model. The data moments are obtained using data on the aggregate firm constructed by summing the variables across all firms in a given year. Income, dividends and investment are all reported as percentages. The sub-sample periods are from 1967/1/1 to 1986/12/31 and 1987/1/1 to 2007/12/31, respectively. $\hat{\Psi}$ denotes a goodness-of-fit measure. The annual economic growth rate, γ, is assumed to be 3%. The pricing kernel is parameterized as

$$\log(M_{t,t+1}) = -(1+b_0)\log(1+\gamma) - b_1(\log(z_t) - \log(z_{t+1})) - b_2(\log(z_t) - \mu_z),$$

where μ_z denotes the mean of $\log(z)$. The applicable pricing kernel for the detrended model equals $M_{t,t+1}$ times $(1+\gamma)$, in order to correctly adjust for the impact of growth. The standard error for the expected real annual return to equity, $E[r]$, is computed using the delta method.

Panel A: Parameters

Sample	Parameter	θ	ρ	σ	λ	b_0	b_1	b_2	$E[r]$	$\hat{\Psi}$
1967 - 1986	Estimate	0.53	0.64	0.032	5.74	0.76	-0.53	-0.27	8.02	3.48
	Standard error	(0.03)	(0.13)	(0.006)	(0.48)	(0.47)	(17.24)	(6.77)	(0.88)	
1987 - 2007	Estimate	0.46	0.67	0.038	3.97	0.32	-0.55	-0.13	5.44	8.17
	Standard error	(0.02)	(0.31)	(0.009)	(0.77)	(0.42)	(23.06)	(7.10)	(0.59)	

Panel B: Moments

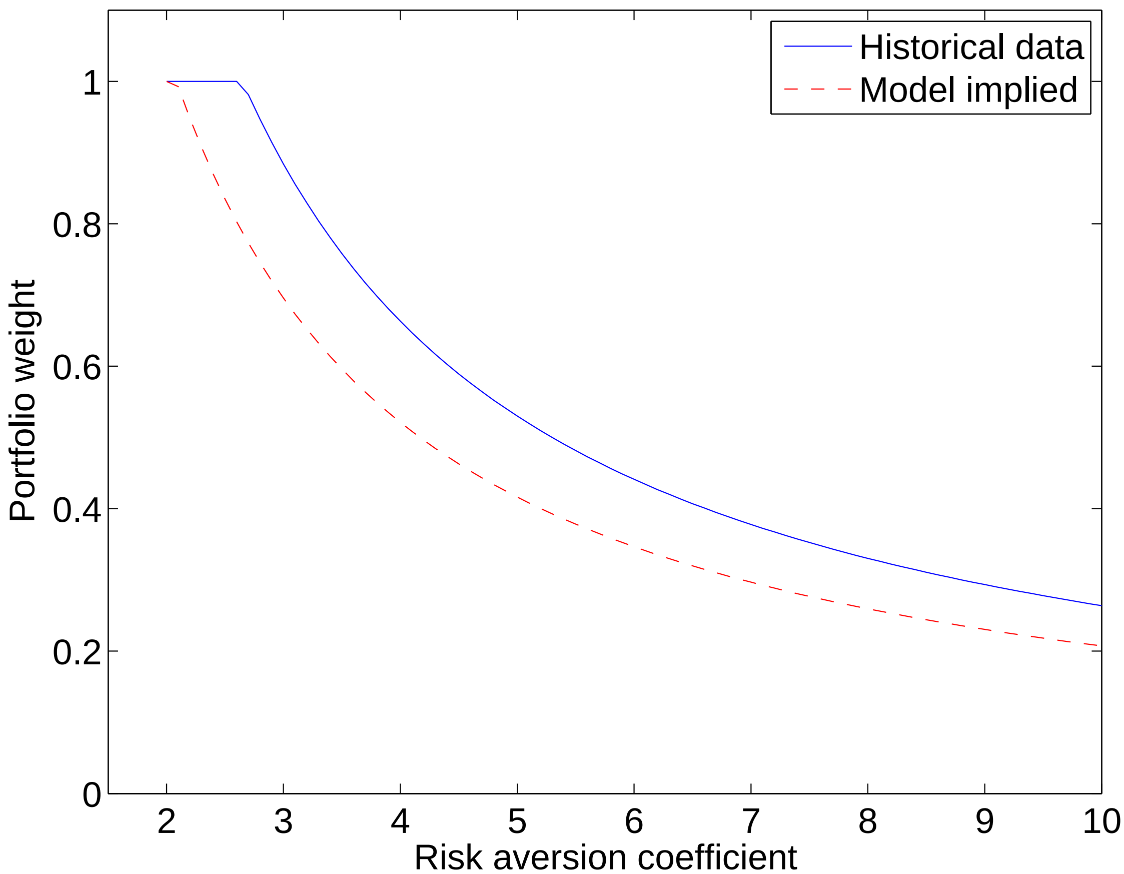
	1967 - 1986		1987 - 2007	
Moment	Data	Model	Data	Model
Mean of income	6.06	6.28	3.98	4.33
Variance of income	1.01	0.88	1.55	0.92
Mean of market-to-book	1.46	1.53	3.10	3.19
Mean of dividends	2.70	2.80	1.89	2.02
Variance of dividends	0.15	0.25	0.16	0.19
Variance of investment	1.43	1.23	1.57	1.65
Autocorrelation of profitability	0.68	0.65	0.69	0.67
Autocorrelation of dividends	0.72	0.85	0.79	0.96

Table 7: Model evaluation

The table reports statistics of interest from the data and the model simulations. The data values are obtained from quarterly data set used in the estimation. The sample period is from 1984/1/1 to 2008/06/30. The model values are obtained using a simulated data constructed with the parameter estimates reported in Table 3. The quarterly growth rate, γ, is assumed to be 0.75%. The value-dividend ratio equals the log of the aggregate market value divided by aggregate dividends. The correlation between dividends, income, and investment are reported after scaling by total assets to induce stationarity.

Statistic	Data	Model
Standard deviation of market return	16.2	12.0
Standard deviation of value-dividend ratio	0.21	0.49
Autocorrelation of value-dividend ratio	0.94	0.88
Standard deviation of dividend growth	0.14	0.12
Autocorrelation of dividend growth	-0.51	-0.16
Correlation between dividends and income	0.44	0.86
Correlation between dividends and investment	0.51	0.52

Figure 1: Optimal portfolio weight for equity



The figure plots the optimal portfolio weight for the stock market as a function of risk aversion for a CRRA utility investor with a horizon of 1 year. The portfolio consists of the aggregate stock market and a risk free bond. The real return on the bond equals 0.5%. The solid line represents the portfolio weights obtained using an expected annual real return to equity equal to its historical average of 6.5%. The dashed line represents the portfolio weights obtained using the model implied annual real return to equity of 5.11%.