Profitability and the Lifecycle of Firms

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Abstract

Using data on private and public firms, this study documents that profitability follows a hump shape over the lifecycle of a firm. Profitability rises with age for young firms, remains elevated, and then declines slowly for mature firms. A dynamic lifecycle model captures the observed age profile of profitability. Investment in product development generates profitability increases for young firms while wage pressures from more productive entrants lead to profitability declines for mature firms. The model generates the lifecycle behavior of financing and growth documented in the literature, even though it contains no financial frictions. It also implies greater sensitivity of financing and growth to age for young firms, a prediction supported by empirical tests. Taken together, these findings indicate that profitability dynamics influence the financing and growth of firms over the lifecycle.

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1 Introduction

Firms are born, live, and die. The literature on firm dynamics highlights the roles of selection, as emphasized by Jovanovic (1982), and survival, as emphasized by Hopenhayn (1992). Another strand of the literature focuses on the role of financial frictions such as costly external finance in understanding firm lifecycle dynamics (see Cooley and Quadrini (2001) and Cabral and Mata (2003)). These studies take the firm's productivity or profitability process as exogenous. Yet, firms spend significant resources on improving their profitability, suggesting that profitability dynamics may play a key role in understanding the financing and growth of firms over the lifecycle.

Using data on private and public firms in the U.K., this study documents that profitability, measured as return on assets, follows a hump shape over the lifecycle. On average, profitability rises with age for young firms, remains elevated for some years and then declines slowly for mature firms. The differences in profitability levels are statistically and economically significant, with average profitability at peak more than three percentage points greater than average profitability for the youngest firms. In addition, the observed age profile of profitability also arises within subsamples for each major industry category.

The observed age profile of profitability may reflect the effects of selection and survival. An examination of changes in profitability over the lifecycle reveals that, on average, young firms realize profitability increases while mature firms realize profitability decreases. This indicates that at least some of the lifecycle dynamics of profitability arises from within-firm changes, and not from systematic differences across firms. Further, a logistic regression of firm exit reveals that profitability has less impact on exit for young firms than for mature firms, suggesting that the increases in profitability for young firms is unlikely to be mainly due to the exit of firms receiving adverse profitability shocks.

Motivated by these findings, this study presents a dynamic firm lifecycle model that features endogenous changes in profitability. Firms in the model generate stochastic quality increases through product development expenditures. An increase in product quality leads to an outward shift in the demand curve for the product, resulting in an increase in profits. On the other hand, the entry of more productive firms pushes up aggregate wages in the economy, putting downward pressure on profits of incumbents.[1] Finally, the model incorporates a survival effect as some firms that realize adverse productivity shocks exit each period. These features combine to generate rich lifecycle dynamics of profitability. These dynamics also influence the financing and growth decisions of firms,

[1]See Foster, Haltiwanger, and Syverson (2008) for evidence on the productivity advantage of entrants.

as firms respond to quality increases by increasing their capital stock to enable them to produce more, and as firms obtain external financing when needed to finance their physical investment and product development.

When calibrated to data on U.K. firms, the model generates the hump-shaped age profile of profitability observed in the data. In the calibrated model, young firms invest heavily in product development expenses, helping generate increases in product quality and profitability. The higher investment in product development by young firms reflects their longer expected lifespan, and the fact that a downward sloping demand curve implies greater increases in firm value from a quality increase for firms with low quality. In comparison, mature firms invest less in product development, and face declining profits due to rising wage pressures.

The higher rate of quality increases for young firms translates into greater capital expenditures, as firms increase their output to take advantage of the rising demand for their products. This leads young firms to grow faster than mature firms. In addition, the combination of high investment in product development andhigh physical investment imply that young firms obtain external finance at much higher rates in the model and pay little or no dividends. As such, the model generates the lifecycle behavior of financing (see DeAngelo, DeAngelo, and Stulz (2006)) and growth established in the literature (see Dunne, Roberts, and Samuelson (1989) and Haltiwanger, Jarmin, and Miranda (2013)). Notably, the model generates the lifecycle behavior of financing, which has been argued as evidence in favor of financial constraints, even though it contains no financing frictions at all. Gomes (2001) and Moyen (2004) present analogous findings that the investment-cash flow regression used as evidence of financial constraints can arise absent financial frictions. In addition, a counterfactual analysis reveals that firm growth would be much slower absent the quality ladder. This result that suggests that difficulties in obtaining quality improvements may help explain the finding in Hsieh and Klenow (2014) that establishments in India and Mexico grow much slower than those in the U.S.

Regression analysis of the simulated data reveal that the model generates a less intuitive prediction relating firm age to finance and growth. Specifically, the model implies that the sensitivity of financing and growth decisions to age is larger for young firms that mature firms. This reflects the fact that the early years of a firm are more formative than later years in the model, reflecting the greater likelihood of quality changes for young firms. This finding can be viewed as analogous to the literature emphasizing the role of early childhood education in human capital formation (see Cunha and Heckman (2007)). It also implies that policy interventions—such as wage and investment subsidies—targeted at young firms can have beneficial effects well after the firms stop receiving such subsidies.

Regression analysis using the data on U.K. firms supports the prediction that the sensitivity of firm growth and financing to age is stronger for young firms than mature firms. This finding provides support for the quality ladder mechanism as the model goes not generate this implication absent it. In addition, the empirical analysis reveals that the sensitivity of age to finance and growth decisions is greater for firms in R&D-intensive industries, where one may expect a quality ladder mechanism to play a greater role. Finally, a logistic regression reveals that, among young firms, age has an effect on whether firms realize profitability jumps, consistent with the model.

Recent studies that highlight the role of innovations in understanding firm dynamics include: Ericson and Pakes (1995), who examine industry dynamics in a setting where firms invest in research and development; Klette and Kortum (2004) and Lentz and Mortensen (2008), who examine firm dynamics in a model in which firms innovate to generate increases in the number and quality of their products; and Luttmer (2011), who uses a model of experimentation to understand the firm size distribution and explain why some firms are long-lived. In comparison, this study uses a quality ladder framework to generate the observed age profile of profitability and match the behavior of financing and growth over the lifecycle.

Among related studies, Albuquerque and Hopenhayn (2004) examine firm dynamics in a model with optimal debt contracts. Bitler, Moskowitz, and Vissing-Jorgensen (2005) find that higher entrepreneurial ownership stakes lead to increased effort and performance. Cagetti and De Nardi (2006) demonstrate that a model of entrepreneurship in the presence of borrowing constraints can generate the observed wealth distribution in the U.S. economy. Angelini and Generale (2008) document that costly external finance explains only a small portion of the effect of firm age on firm size. Aghion, Blundell, Griffith, Howitt, and Prantl (2009) examine the effect of entry on innovation and productivity growth of incumbents. Chen, Miao, and Wang (2010) examine the effect of entrepreneurial non-diversifiable risk on the financing and investment decisions of firms. Huynh and Petrunia (2010, 2015) study how financial conditions affect firm growth and survival. The internal governance model of Acharya, Myers, and Rajan (2011) generates a lifecycle for dividend payments from the IPO of the firm. And Haltiwanger, Jarmin, and Miranda (2013) examine the effect of firm age and size on growth, and argue in favor of further research on how earnings of firms vary with age.

This study is organized as follows. Section 2 presents the data. Section 3 examines the age profile of profitability in the data. Section 4 presents the model. Section 5 discusses the model calibration and analyzes data generated by simulating the model. Section 6 presents regression evidence on firm growth and financing using data on U.K. firms and Section 7 concludes.

2 Data

The data set used in the study is obtained from the Amadeus database maintained by Bureau van Dijk. This data set provides balance sheet and income statements for listed and unlisted firms in many European countries from 1997 to 2008. The analysis uses data on firms from the U.K. to avoid any cross-country differences. Focusing only on U.K. firms helps mitigate any cohort effects that may arise from the introduction of the euro in continental Europe. Further, the accounting regulations in the U.K. require all firms to file annual accounts at the Companies House. These legally required filings provide the source data for the Amadeus data set.[2] One limitation of the data set is that it does not include information on product development expenses such as research and development and advertising.

The data set includes the year and month of incorporation of the firm. This enables a more accurate measure of firm age in the data than compared to what could be obtained using data sets such as Compustat, where age is typically measured from the date of the initial public offering. In addition, the data set includes observations of firms in their earliest years, enabling a more detailed analysis of lifecycle effects than would be possible using data sets of mostly public firms. The Census Bureau's Longitudinal Business Database, used by Haltiwanger, Jarmin, and Miranda (2013), among others, also provides accurate measures of firm birth and includes the early years of firms, but does not include accounting data on profitability or financing measures.

One limitation of Amadeus is that it includes information on firms that exit only during the last 5 years of the sample. That is, the data set includes firms that exited from 2004 to 2008, but excludes those who exited prior to 2004. This leads to a potential survival bias in the analysis that will be addressed using a two-step Heckman selection estimator (see Heckman (1979)).

[2] The source data for the U.K. data is the same as that for the FAME database, which is also maintained by Bureau van Dijk. Brav (2009) provides a detailed description of how Bureau van Dijk constructs the FAME database.

2.1 Sample

The sample period extends from 1997 to 2008, as the data set contains few observations in the years prior to 1997. Firms with missing values for total assets, year of incorporation, or revenue are excluded from the sample. Firms with less than 5 employees are also excluded, partly to eliminate self-employed individuals that have chosen to incorporate as a firm. The sample excludes any firms with total assets less than 100,000 pounds. The sample also excludes observations with accounting periods other than one year, and observations of financial firms, as identified by 2-digit SIC codes. All observations are rescaled to take into account different units of observations for different firms in the data set.[3]

The study uses the following variable definitions. Age is measured from the month of incorporation to the month-end date for the accounting statements. As such, age is measured in years and months. Firm size equals the log of total assets. Sales growth is defined as the growth rate of operating revenues. Profitability is measured as return on assets, which equals operating profits before interest and depreciation divided by average total assets over the year. This measure captures the operational strength of the firm. Firm exit is measured based on the legal status of the firm. A firm that does not have a legal status of "active" is considered to have exited. Physical investment is measured as the book value of fixed assets minus lagged fixed assets, divided by lagged fixed assets. This measure captures the growth in fixed assets employed by the firm. All variables except age and size are Winsorized at the 1 percent level to reduce the impact of outliers.

One cannot observe when firms realize quality increases, a feature of the model presented in Section 4. As such, a profitability jump variable is constructed to possibly identify firms that realized quality increases. The profitable jump variable is constructed cross-sectionally, with a firm considered to have realized a profitability jump if it's average return on assets from 2004 to 2008 exceeded its average return on assets over the preceeding years by 10 percentage points. Comparing differences in average return on assets across the late and early years of the sample focuses on the permanent nature of profitability increases arising from quality increases and helps avoid misclassifications arising from transient shocks to profitability. In addition, a corresponding profitability drop dummy is constructed to capture firms that face a drop in average return on assets.

This study constructs two measures of external financing during a year using balance sheet

[3]Amadeus reports most observations of firms in U.K. in units of pounds. But, some firms have values reported in thousands of pounds and a few firms have values reported in millions of pounds.

data on the financing employed by firms. A stock issuance dummy variable equals one if the firm's contributed capital was greater than last period's contributed capital plus 2 percent. Such an increase could occur only if the firm obtained additional equity finance during the year, on net. The external financing dummy variable equals one if the sum of the firms contributed capital, debt, and bank loans was greater than the corresponding last period value plus 2 percent. Such a change would reflect a firm obtaining either additional equity or debt finance during the year and provides a broader view of whether the firm obtained external finance that year than the stock issuance dummy. These measures enable an examination of how the use of external financing varies over the lifecycle of the firm.

2.2 Summary statistics

Table 1 presents summary statistics from the data. The table reports statistics for all firms, firms grouped into three age terciles, and firms in R&D-intensive industries. The cut-off values for the age terciles equal 10 years and 9 months, and 23 years and 4 months. Splitting the sample by age terciles helps identify whether age effects differ across age groups, as implied by the model. Further, focusing on firms in R&D-intensive industries enables one to look at a subsample for which product development, a key feature of the model, would presumably be more important. Industry R&D-intensity is measured using data on R&D expenditures by U.S. firms from Compustat and a firm is considered to be in a R&D-intensive industry if it's 2-digit industry code SIC equivalent reported by Amadeus equals either 28, 35, 36, 38, 73 or 87.[4] This follows the research design of Rajan and Zingales (1998), who measure industry financial dependence using data on U.S. firms.

The summary statistics demonstrate a marked effect of age on firms' policies. On average, young firms have higher sales growth, return on assets, and investment rates than mature firms. Young firms also obtain external financing at a higher frequency than mature firms. This difference is particularly notable for equity issuance. The differences in the mean values across the young and mature firm subsamples are statistically significant at the 5 percent level.

The summary statistics also reveal that about 1/7th of the firms realize a profitability jump. A much greater fraction of young firms realize these profitability jumps while only a small fraction of mature firms do so. However, this is at least in part driven by the notable reduction in the volatility of profitability with age, as can be seen in the corresponding drop in the profitability

[4]These industries include pharmaceuticals, computer software, computer hardware, professional services and electronics.

drop dummy variable across age groups. In comparison to the entire sample, a greater fraction of firms in R&D-intensive industries also realize profitability jumps.

The summary statistics indicate that the growth and external financing decisions of firms vary systematically over the firm's lifecycle, consistent with the findings in the literature. The next section examines profitability dynamics over the lifecycle.

3 Profitability over the lifecycle

This section documents the key finding that profitability—measured by return on assets—follows a hump shape over the lifecycle of firms. It examines average profitability levels by age for all firms, firms in major industries, and by survival status of the firm. In addition, it also examines within-firm differences in profitability for firms in the sample. To the best of my knowledge, existing studies have not systematically examined how the profitability of firms varies with age.

3.1 Profitability levels

Figure 1 plots mean return on assets as a function of age for all firms in the sample, where age is rounded to years. The dashed (blue) lines represent the 95 percent confidence interval around the estimated sample means. The figure demonstrates that average return on assets increases up to age 10 (with much of this increase occuring before age 5), remains at or near this level until age 20, and declines slowly thereafter. In statistical terms, firms between ages 10 and 20 have significantly higher return on assets than new firms or firms older than 20 years. The figure demonstrates that average profitability follows a hump shape over the lifecycle of firms.

The age profile of profitability remains robust to changing the Winsorization threshold to 2.5 percent and to computing return on assets by normalizing by total assets at either the beginning or end of the year, instead of normalizing by average total assets as in the above figure. The age profile of profitability also remains basically unchanged when the firm-year observations are weighted by their log assets. In addition, incorporating year fixed-effects has no effect on the observed age profile of profitability.

3.2 Profitability levels by major industry groups

Figure 2 plots mean return on assets as a function of age for firms in selected major industry groups. Panels A, B and C, respectively, plot the results for firms in the manufacturing, services, and retail and wholesale trade sectors, where firms are classified into sectors based on 2-digit SIC codes. Panel D plots the results for firms in R&D-intensive industries, where R&D intensity is measured at the 2-digit SIC code level using Compustat data on U.S. firms.[5]

The figure indicates that one obtains a similar age profile of profitability for firms in the manufacturing and service sectors. In both sectors, mean return on assets rises until about 10 years, remains elevated for some years, and then declines slowly. Firms in retail and wholesale trade exhibit a somewhat different age profile of profitability, with profitability peaking at a younger age. Turning to firms in R&D-intensive industries, one again finds the hump-shaped age profile of profitability observed for all firms. These findings indicate that this age profile of profitability is a consistent feature of the data. Sections 4 and 5 attempt to understand the observed age profile of profitability through the lens of a model that features increases in product quality and wage pressure from new entrants.

3.3 Selection

The observed age profile of profitability may reflect the effect of selection. Selection occurs when firms that exit are systematically different from those that survive. In the seminal Jovanovic (1982) model, selection occurs through differences in mean productivity across firms. Selection can drive the observed increase in return on assets for young firms if the young firms that exit have systematically lower return on assets than those who survive. One approach for evaluating whether selection matters is to examine differences in profitability within firms. As selection pertains to differences in profitability across firms, if the increase in return on assets observed in the data came solely from selection, one would not expect to see any changes in return on assets within firms.

Figure 3 plots the mean change in firms' return on assets from age t to $t + 1$ as a function of age, t. The dashed (blue) lines represent the 95 percent confidence interval around the estimated sample means. The figure demonstrates that firms with age less than or equal to 5 years generate profitability increases, on average. These profitability increases are statistically and economically significant, with a cumulative increase in return on assets of more than 3 percentage points over

[5] These firms are also included in the samples for either the manufacturing sector or the service sector.

this period. Thus, much of the increase in average return on assets shown in Figure 1 arises within firms, indicating that selection among firms does not explain this rise. Further, average changes in return on assets become negative, though for the most part insignificant, for firms older than 20 years. This indicates that the decline in profitability for mature firms also does not arise from selection. However, median changes in return on assets are about zero for all ages, indicating that the average return on assets increases obtained by young firms reflect profitability increases at some, but not all, firms.

Another way of examining the effect of selection is to examine differences across surviving and exiting firms. Figure 4 plots the age profile of profitability for firms categorized by whether they survived through the end of the sample: Panel A plots the age profile for surviving firms and Panel B plots the age profile for firms that exited. The figure demonstrates that selection does have an impact, as exiting firms have a lower mean mean return on assets prior to exit than surviving firms, leading to a small upward shift in the mean return on assets for surviving firms. However, the overall age profile remains mostly unchanged from Figure 1, indicating that selection, by itself, cannot account for the observed age profile of profitability.

3.4 Survival

The effect of survival, as in Hopenhayn (1992), may also explain the observed age profile of profitability. Young firms that receive negative profitability shocks may exit, leading to increases in observed mean return on assets for surviving young firms. On the other hand, mature firms with high return on assets may exit through mergers and acquisitions, leading to declines in mean return on assets among surviving firms. The age profile of profitability for firms that exited, discussed above in Panel B of Figure 4, shows that even among firms that exited, average return on assets increases for young firms. This provides tentative evidence that survival cannot account for all the increase in average profitability for young firms observed in the data.

In order to further understand survival dynamics in the data, Table 3 presents the results of a regression of firm exit on age, return on assets and controls. The regression is carried out on data from 2004 onwards, as firms that exited in prior years are excluded from the sample. The results are reported for all firms and for firms grouped by age terciles.

The findings indicate that younger, less profitable are more likely to exit. This suggests that survival effects account for some of the profitability increase for young firms observed in the data. However, the effect of return on assets on survival is strongest for mature firms, indicating that profitability has less impact on exit for young firms, thus mitigating the survival effect. Further, the exit rate for firms aged 1 to 5 years equals 3.2%, compared to an overall exit rate of 1.8%.[6] Thus, in order for survival effects to fully explain the gap in mean return of assets for firms aged 1 to 5 of 7.9% and the mean return on assets for all firms of 9.2%, one would require the return on assets of exiting young firms to be sharply below that of surviving firms, contrary to the regression evidence noted above. Overall, these findings suggest that survival plays only a partial role in understanding the observed age profile of profitability.

The literature on mergers and acquisitions indicates that the decline in profitability for mature firm is unlikely to occur due to the exit of high return on assets firms via such transactions. Theoretical models, such as Jovanovic and Rousseau (2008) imply that firms with low Tobin's Q are likely to exit via acquisitions; such firms will also have low return on assets. Studies such as Harford (2005) and Warusawitharana (2008) confirm this prediction by demonstrating that firms with low return on assets are more likely to be targets of acquisitions.

The results also indicate that cash holdings have a strong negative effect on exit, consistent with economic intuition. Assuming that cash holdings have little effect on firm growth, this will help provide identification of the subsequent Heckman selection regressions presented in Section 6.

4 Model

This study interprets the observed age profile of profitability through the lens of a dynamic lifecycle model of the firm. The model economy consists of a large number of heterogenous firms that produce differentiated products. The quantity produced by a firm varies with its productivity, capital stock and labor input. The price of its product depends on the product quality, with higher quality products earning a higher price. The profitability of the firm is a function of both its productivity and product quality. Firms invest resources on physical investment and quality improvements in order to maximize the present value of dividends over their lifetimes. The firms are owned by a representative household who supplies a fixed quantity of labor.

[6] The exit rate for firms of 1.8% in the Amadeus data is smaller than the exit rate of 5.5% obtained by Lee and Mukoyama (2015) using establishment level data from the U.S. The higher exit rate for estalishments is perhaps unsurprising given the fact that such exits include not only the exit of entire firms, but also the closure of individual establishments by multi-establishment firms.

4.1 Output, demand, and profits

Each firm, indexed by j, uses capital, K_j, and labor, L_j, to produce a single product using a Cobb-Douglas production function.[7] The model features a vintage productivity effect, with the labor augmenting productivity of an existing firm of age a_j at time t, denoted by μ_{t-a_j}, assumed to be fixed by its vintage $t - a_j$.[8] Firms with a later vintage have a higher average productivity level, reflecting the higher productivity of entrants documented by Foster, Haltiwanger, and Syverson (2008). The output of a firm of age a at time t is given by

$$Y_j(K_j, L_j, a_j) = \mu_{t-a_j}^{1-\alpha} \tilde{z}_j K_j^{\alpha} L_j^{1-\alpha}, \tag{1}$$

where $\tilde{z}_j$ denotes transient shocks to productivity, and α denotes the capital share of output. This specification implies that the firm employs the optimal capital-to-labor ratio each period.

The output of all firms are combined into a consumption aggregate, C_t, using a quality-weighted Dixit-Stiglitz aggregator with constant elasticity of substitution (Dinopoulos and Thompson (1998) employ a similar aggregator).

$$C_t = \left(\int_j q_j(n) Y_j^{1-\nu} dj \right)^{\frac{1}{1-\nu}},$$

where $q_j(n)$ denotes the quality level of good j, with higher quality indices n yielding higher utility per unit of output. Normalizing the aggregate price level to 1, the price charged for good j is given by

$$P(Y_j) = q_j(n) Y_j^{-\nu} C_t^{\nu}, \tag{2}$$

where the mark-up ν depends on the elasticity of substitution between two goods. This specification implies that a product of higher quality commands a higher price for the same quantity, reflecting the higher utility that consumers derive from its consumption.

Let w_t denote the aggregate wage level. The gross profit of a firm of quality $q_j(n)$ is given by

$$\Pi_j(K_j, a_j, \tilde{z}_j; q_j(n)) = \max_{L_j} P(Y_j) Y_j - w_t L_j, \tag{3}$$

[7] For notational simplicity, the study omits time t subscripts from all firm specific variables. Time subscripts are shown for aggregate variables such as the vintage productivity term, μ_t.

[8] Solow, Tobin, von Weizsäcker, and Yaari (1966) present an early model with such a vintage specific productivity term. For simplicity, I assume no productivity growth for incumbents. Allowing the productivity of all existing firms to grow at a constant rate has no effect on the firm's optimal policies.

where the price, $P(Y_j)$, and output, Y_j, are given by equations (2) and (1), respectively. Some algebra yields the following expression for profits:

$$\Pi_j(K_j, \tilde{z}_j, a_j; q_j(n)) = (1-\theta)\left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\theta}} \left[q_j(n) C_t^{\nu} (\mu_{t-a_j}^{1-\alpha} \tilde{z}_j)^{1-\nu} K_j^{\alpha(1-\nu)}\right]^{\frac{1}{1-\theta}}, \tag{4}$$

where $\theta = (1-\alpha)(1-\nu)$. The mark-up in the pricing equation (2) leads to decreasing returns to scale in the profit function, implying a bounded optimal firm size. In comparison, studies that follow Lucas (1978) generate an optimal firm size based on the assumption of a non-reproducible factor such as managerial talent.

4.2 Balanced growth path

The above profit function trends with the consumption aggregator, C_t, and the wage level, w_t. Assuming that the vintage productivity term μ_t grows at a constant rate g, Appendix A demonstrates that the aggregate capital stock, consumption and wages growth at the same rate g.

Thus, one can translate the above profit function into a stationary function by dividing through by a trend growth variable X_t that is proportional to C_t, w_t and μ_t. Using lower case letters to denote detrended variables, let $\pi_j = \frac{\Pi_j}{X_t}$, $k_j = \frac{K_j}{X_t}$ and $c^* = \frac{C_t}{X_t}$. Dividing through by X_t and rearranging terms, one obtains the following:

$$\pi_j(k_j, \tilde{z}_j, a_j; q_j(n)) = (1-\theta)(\theta)^{\frac{\theta}{1-\theta}} \left[q_j(n)\left(\frac{\mu_{t-a_j}}{w_t}\right)^{\theta} c^* \tilde{z}_j^{1-\nu} k_j^{\alpha(1-\nu)}\right]^{\frac{1}{1-\theta}}, \tag{5}$$

where we have detrended consumption and capital, and gathered together the terms involving vintage productivity and the wage.

The above equation highlights the effect of new entrants on the profits of incumbents. The entry of more productive firms drives up wages in the economy. As the output productivity of incumbents remains fixed by vintage, the $\frac{\mu_{t-a_j}}{w_t}$ term declines with age, putting downward pressure on profits, *ceteris paribus*. One can further simplify the above profit function by noting that $\frac{\mu_{t-a_j}}{w_t} = \frac{\mu_t}{w_t}(1+g)^{-a_j}$ and that $\frac{\mu_t}{w_t}$ is a constant as both μ_t and w_t grow at the same rate g. Thus, one obtains the following detrended profit function:

$$\pi(k_j, \tilde{z}_j, a_j; q_j(n)) = c\left[q_j(n)(1+g)^{-a_j\theta} \tilde{z}_j^{1-\nu} k_j^{\alpha(1-\nu)}\right]^{\frac{1}{1-\theta}}, \tag{6}$$

where c is a constant that gathers together terms involving θ, c^* and $\frac{\mu_t}{w_t}$. The subsequent analysis employs this detrended profit function.

4.3 Endogenous product quality

The above discussion examines the effect of firm age on profitability while treating product quality, $q_j(n)$, as given. However, firms spend significant resources on product development. These include not only research and development expenditures, but also expenditures such as advertising that can potentially increase the demand for a good. Related, Foster, Haltiwanger, and Syverson (2012) find that variation in the size of plants mainly reflects demand-side fundamentals and argue in favor of a demand accumulation process at the plant level. And Aghion, Bloom, Blundell, Griffith, and Howitt (2005) investigate the effect of competition on product development by firms.

In the model, firms have product quality levels $q_j(n)$ indexed by n. New firms enter the economy with a quality index of 1. Each period, a firm spends resources on product development, the detrended value of which is denoted by r. These product development expenses are subject to a non-negativity constraint. If the firm's product development was successful it realizes an increase in product quality, with probability of success increasing in r.[9] Formally, the evolution of product quality is given by the following equation:

$$\begin{aligned} q'_j &= q_j(n+1) \text{ with probability } p(r), \text{ or} \\ &= q_j(n) \text{ with probability } (1-p(r)), \end{aligned} \tag{7}$$

where q'_j denotes the next period quality level of a firm with current quality level $q_j(n)$.

An increase in product quality results in a proportional increase in the profits of the firm, as in the quality ladder literature (see Grossman and Helpman (1991) and Aghion and Howitt (1992)). Formally, the quality levels $q_j(n)$ are given by

$$q_j(n) = (1+\gamma)^{n-1}, \tag{8}$$

where γ is a parameter that determines the rate of increase in profits from a quality increase.

[9]This setup reflects the basic structure of the quality ladder literature. Klette and Kortum (2004) and Lentz and Mortensen (2008) presents a different approach where each firm can increase the number of products they sell.

The success probability of generating a quality increase is parameterized as follows:

$$p(r) = 1 - \exp\left(-b\frac{r}{q_j(n)}\right), \tag{9}$$

where b is a parameter that influences the success rate.[10] The above exponential function provides a parsimonious approach to modeling success rates. It implies that the marginal probability of success decreases as r increases.

The scaling of the success probability with the current quality level, $q_j(n)$, ensures that as quality rises firms need to spend additional resources to obtain further quality increases. Such an assumption is commonly used in the quality ladder literature and helps limit exponential increases in product quality at firms. This assumption captures the idea that the product development efforts required to obtain a firm-wide increase in product quality would be greater for a large firm that has already achieved a high quality level than for a small firm with a low quality level. For example, one could compare the resources spent by Microsoft on developing the latest Windows products with those spent by a small software company on developing their latest product.

4.4 Other firm policies

The investment and entry and exit decisions in the model follow standard assumptions.

4.4.1 Investment

Each period, firms can invest in new capital. Denote new investment by i_j. The detrended next period capital of a firm is given by

$$k'_j(1+g) = k_j(1-\delta) + i_j,$$

where δ denotes the depreciation rate.[11] In addition, firms face a quadratic adjustment cost of investment, given by $\lambda\frac{i_j^2}{2k_j}$. This adjustment cost is commonly used in the investment literature (see Hayashi (1982)) and helps limit the volatility of investment.

[10] Warusawitharana (2015) uses a related exponential specification to study the contribution of R&D investment to firm value.

[11] The $(1+g)$ term arises from the division by X_t to detrend the capital accumulation equation.

4.4.2 Entry and exit

A fixed cost of operating each period, f, implies that firms will exit if their value falls below a certain threshold. Firms exit each period after they realize their idiosyncratic shock $\tilde{z}$. Exiting firms sell their capital at a discounted price s. Firms decide to exit optimally if their continuation value is lower than the value they would obtain by selling their capital.

Potential entrants face a fixed entry cost, ϕ. New firms enter the economy with quality index 1. They have an initial capital stock k_0 and begin operations immediately. In the steady state simulations, the rate of entry is assumed to equal the rate of exit.

4.5 Firm value

The value of the firm is given by the solution to a Bellman equation, with the firm's capital, quality, age and idiosyncratic productivity as state variables. Firms optimize over product development expenses and physical investment. Omitting the firm subscripts j, one can write the detrended value function of the firm as:

$$\begin{aligned}
v(k, q(n), a, \tilde{z}) &= \max_{i,r,k'} \; d + \beta(1+g)E_z\left[p(r)v_c(k', q(n+1), a+1, \tilde{z}')+\right. \\
&\quad \left.(1-p(r))v_c(k', q(n), a+1, \tilde{z}')\right], \\
s.t. \quad (1+g)k' &= k(1-\delta)+i, \\
p(r) &= 1-\exp\left(-b\frac{r}{q(n)}\right), \\
\text{and} \quad d &= (\pi(k, \tilde{z}, a; q_n) - f - r) - i - \lambda\frac{i^2}{2k},
\end{aligned} \tag{10}$$

where $v_c(k', q(n), a+1, \tilde{z}')$ equals the continuation value of the firm. The possibility of exit implies that the continuation value is given by

$$v_c(k, q(n), a, \tilde{z}) = \max\{v(k, q(n), a, \tilde{z}), sk\}.$$

The $(1+g)$ terms appear in the discount factor to take into account the effect of growth, as in Eberly, Rebelo, and Vincent (2008).

In economic terms, the above Bellman equation states that the value of a firm equals the dividend payment plus the expected future value of the firm. A firm that does not generate sufficient internal funds to finance all their expenditures will obtain external finance, indicated by a negative value for dividends. The expected future value of the firm takes into account that the firm's age increases by one each year, and that the firm may realize an increase in the quality index with probability $p(r)$. The discount rate remains unchanged over the lifecycle, reflecting the finding of Moskowitz and Vissing-Jorgensen (2002) that private firms generate similar returns as public firms. Each firm chooses its physical investment and product development expense to maximize value.

4.6 Optimal product development expenses

Firms trade off the cost of product development expenses with the higher probability of an increase in the quality index, which increases firm value. This increase in firm value with the quality index reflects both the ability of the firm to sell current output at a higher price, and the ability of the firm to invest and increase production in the future to take advantage of the shift in the demand curve.

The following proposition establishes the optimality condition for product development expenses:

Proposition 1 *The first order condition for product development implies that*

$$\begin{aligned} \textit{marginal cost of funds} &= \textit{marginal benefit of product development expenses} \\ \Rightarrow \qquad 1 &= \beta(1+g)E_z\left[\frac{v(k', q(n+1), a+1, \tilde{z}') - v(k', q(n), a+1, \tilde{z}')}{q(n)}\right] b(1-p(r)). \end{aligned}$$

Proof. See Appendix B. ■

The marginal benefit from product development expenses rises with the expected increase in firm value from a quality increase. The $(1-p(r))$ term implies that an increase in the success probability $p(r)$ lowers the marginal benefit, ensuring an interior solution for product development expenses, subject to the non-negativity constraint, $r \geq 0$.

4.7 Equilibrium

The steady-state equilibrium in the model is characterized by the following conditions:

1. The joint distribution of firm age, quality and capital stocks remains invariant over time as firms exit and new firms enter.

2. The mass of firms in the economy is such that the expected value of a firm upon entry equals the cost of entry.

3. The aggregate goods and labor markets clear.

These conditions are verified in the subsequent calibration and simulation of the model.

4.8 Discussion of lifecycle mechanics

The mechanisms underlying the model are intuitive. The evolution of profitability with firm age drives the lifecycle in the model. As seen in equations (5) and (6), competition from more productive entrants drives up wages in the economy and puts downward pressure on the profits of incumbents. Product development generates stochastic increases in the quality level of firms' products—see equation (7), (8) and (9)—boosting profits of incumbent firms. The combination of these two features generate profitability dynamics over the lifecycle, which influences firms' growth and financing decisions.

Young firms enter the economy with quality index 1. These firms can realize a large jump in firm value through a quality increase, leading them to expend significant resources on product development. Firms increase physical investment following a quality increase as the increased demand for the good raises the optimal capital stock. This leads young firms to seek external financing and attempt to grow rapidly. For the youngest firms, increases in product quality can help overcome the effect of competition from new entrants, potentially resulting in profitability rising with age. As quality increases and firms grow, they reduce spending on product development, and the effect of competitive pressures on wages eventually dominates the effect of quality changes, resulting in a steady decline in profits for mature firms.

5 Model simulation and analysis

This section examines the implications of the above model using simulated data from a model calibration. The procedure used to generate the simulated data set is detailed in Appendix C.

First, it examines whether the model can generate the age profile of profitability observed in the data. While this may not be surprising as the calibration is aimed, in part, at matching this profile, it nonetheless provides a useful quantitative validation of the model.

Next, it derives additional testable predictions on the strength of age effects across young and mature firms using regression analysis of simulated data. This regression analysis of simulated data is necessary as the stochastic nature of the lifecycle and the complexity of the model makes it difficult to generate explicit theoretical propositions relating age to firms' policies. The regression analysis with the simulated data will be subsequently repeated using the actual data on U.K. firms (see Section 6). Whited (2006), and Bloom, Bond, and van Reenen (2007), among others, use the same approach of comparing results from a simulated model with the results from actual data.

Last, this section presents findings from simulations using counterfactual model experiments. These experiments help further understand the mechanisms in the model and highlight the effect of potential policy changes that benefit young firms. They also help shed light on the role of the quality ladder mechanism in driving firm growth.

5.1 Calibration

The model parameters are calibrated by either using values estimated or commonly used in the literature, or by matching selected moments in the simulated and actual data.

The following parameters are set to values commonly used or estimated in the literature. The capital share of output, α, equals 0.33. The price markup parameter ν equals 1/7, implying a price elasticity of demand of 6. These imply that $\theta = 0.574$. The depreciation rate is set at an annual rate of 10 percent. The discount rate $\beta = 0.939$, corresponding to an annual real return to capital (equity) of 6.5 percent. The growth rate of the trend variable, g, is set equal to 1.5 percent. The adjustment cost parameter λ equals 4. This value is close to the estimate of 3.5 obtained by Eberly, Rebelo, and Vincent (2008). The resale value of capital is set at 0.8, in between the estimate of 0.6 obtained by Hennessy and Whited (2005) and the estimate of 0.95 obtained by Cooper and Haltiwanger (2006).

The remaining parameters are calibrated to match the following data moments: the mean and variance of profitability; profitability at entry; profitability at peak; profitability of mature firms; mean exit rates; mean investment; and the mean and the 25th and 75th percentiles of the age distribution. The autocorrelation and standard deviation of productivity shocks are set to 0.7 and 0.5, respectively. The constant term in the profitability equation, (6), is set to 0.05 to generate a mean profitability of entrants of 8 percent, close to the data. Given the other parameters, the fixed cost of operations is set to generate a mean exit rate of 2.8%.

The parameters for the quality ladder mechanism are set to match the observed age profile of profitability. The proportional increase in the quality level, γ, equals 0.0333. Given the θ value specified above, this translates to an 8% increase in profits from a successful quality increase. The quality index takes values from 1 to 15. The parameter that influences the success rate b equals 77. These values help match profitability at peak, and the mean and percentiles of the age distribution.[12] If the quality ladder parameters were much weaker, one would obtain a simulated data set with no quality increases, and if they were much stronger, firms would obtain profitability increases for a much longer period following entry.

Table 2 compares the matched moments from the data with the corresponding values obtained from simulating the calibrated model. As the table indicates, the calibrated model does a good job of matching profits, both for all firms, and for firms at specific points of the lifecycle. The model also generally succeeds in matching the observed age distribution, though the 75th percentile of age is lower in the data than in the model, possibly reflecting the effect of unobserved heterogeneity across firms. The model also succeeds in coming close to the mean investment rate observed in the data. The model is less successful at matching the volatility of profits and the exit rate, with the simulated model exhibiting lower volatility of profits and higher exit rates than observed in the data. In part, this reflects the fact that profitability shocks are the main driver of exit in the model, which implies that exit rates are increasing in the volatility of profits.

5.2 Age profile of profitability

Figure 5 presents average profitability as a function of age from the simulated data set. Profitability is derived from equation (6) as

$$c\left[q_j(n)(1+g)^{-a_j\theta}\tilde{z}_j^{1-\nu}\right]^{\frac{1}{1-\theta}}.$$

Thus, both the current quality of the product, $q_j(n)$, and its output productivity level $\tilde{z}_j$ affect the profitability of the firm. An alternate approach that provides a broader definition that incorporates both scale effects and the fixed cost would be to define profitability as $\frac{\pi(k_j,a_j,\tilde{z}_j;q_j(n))-f}{k_j}$. This measure has the drawback that changes in the ratio of fixed costs to the capital stock has a

[12]The mean quality index from the simulated data set equals 7.30.

substantial effect on profitability, particulary for rapidly growing young firms. In comparison, the definition used in the study provides a more focused measure of profitability, as it is influenced only by changes in age, product quality and output productivity.

The figure demonstrates a strong lifecycle effect on profitability for firms. Young firms obtain sharp profitability increases, partly reflecting quality increases from successful product development. As firms age, they realize fewer endogenous quality increases. Beyond a point, the competitive pressures from the productivity advantage of new entrants dominates the effect of quality increases. This leads to a slow decline in average profitability. The relatively wide confidence intervals associated with mean profitability reflect the volatility of profitability shocks in the simulated data.

The key mechanism driving the age profile of profitability in the calibrated model is that young firms spend more resources on product development, resulting in more quality increases. What drives this higher investment in product development in the model? First, young firms have a higher expected lifespan, resulting in a greater increase in firm value from a quality increase. Second, the decreasing returns in the profit function imply that small, low-quality firms obtain greater increases in firm value from a quality increase. Third, the vintage capital feature of the model implies that older firms are less profitable than younger firms of the same quality level, resulting in lower product development expenditures.

5.3 Regression analysis of simulated data

This section uses regression analysis to examine whether the model generates the lifecycle behavior of financing and growth documented in the literature. In addition, it also examines whether the sensitivity of financing and growth to firm age differs across young and mature firms. As these features of the simulated data set were not targeted in the model calibration, this analysis helps yield additional testable predictions from the model.

Table 4 presents the results of firm growth regressions on age and controls. Panels A and B, respectively presents linear panel regressions for sales growth and investment. The availability of actual data constrain the controls used in the regressions.[13] All the regressors are statistically significant at the 5 percent level, reflecting the importance of the regressors in the model.

The results demonstrate that younger firms in the model grow at a faster rate than older firms,

[13] For instance, the Amadeus data on firms does not include firm value as most of the firms are privately held. This makes it impossible to construct Tobin's Q with the actual data.

reflecting the findings of Dunne, Roberts, and Samuelson (1989) and Haltiwanger, Jarmin, and Miranda (2013).[14] In addition, the results also indicate that the effect of age on firm growth is much stronger for young firms than mature firms. This reflects the fact that young firms obtain product quality increases at a faster rate than mature firms. Thus, both product quality and profitability changes rapidly with age for young firms in the model, leading to rapid changes in growth.

This effect arises as young firms obtain quality increases at a higher rate than mature firms who, for the most part, will have already achieved a high quality level. Thus, changes in age translate to bigger changes in product quality and profitability for young firms, implying that age will have a greater effect on the growth of young firms than mature firms. This suggests that the early years of a firm play an important formative role in determining eventual outcomes for firms.

Panels A and B of Table 5 present the results of logistic regressions for whether firms obtain external financing or realize quality increases, respectively, from the simulated data set. The regressors include age and various controls. As before, the control variables are significant in most of the specifications.

The findings reported in Panel A indicate that young firms obtain external financing at a higher frequency than mature firms, consistent with the findings in the literature (see DeAngelo, DeAngelo, and Stulz (2006)). Notably, this arises even though the model contains no financial frictions. The above finding reflects the fact that young firms have a much larger demand for financing physical investment and product development than mature firms and thus require more external financing. In contrast, mature successful firms become self-financing as they generate high profits and have lower investment rates and product development expenses. The results also indicate that the sensitivity of external financing choices to age is much greater for young firms than mature firms. As before, this increased sensitivity reflects the fact that the early years of a firm are more formative in the model.

The regression results in Panel B indicate that young firms obtain quality increases at a faster rate than mature firms. In addition, the sensitivity of age to quality increases is much greater for young firms than mature firms. The following factors contribute to the greater investment in product development by young firms: a higher expected lifespan for young firms; decreasing returns in profits that imply small, low-quality firms obtain greater increase in firm value from a quality increase; and lower profitability levels for mature firms due to new entrants pushing up wages in

[14] Clementi and Palazzo (2015) examine the effect of the faster growth of young firms on aggregate fluctuations in a setting with endogenous entry and exit.

the economy.

In unreported results, a full interaction specification reveals that the differences in age effects between young and mature firms are statistically significant. Repeating the analysis for firms with age below the 33.3th percentile generates similar findings for the investment, quality increase and external financing regressions; the age effect for sales growth becomes insignificant in this subsample due to a strong correlation between firm size and age for these firms in the simulated data set.

The results obtained from the analysis of the simulated data set remain robust to modest changes in the calibrated parameter values. Although the estimated coefficients change, the main findings remain. Young firms obtain more frequent quality increases, which result in higher sales growth and investment rates. High product development expenses and physical investment lead young firms to obtain external finance more often. Further, the effect of age on these policies is more pronounced for younger firms.

5.4 Model experiments

This section reports the age profile of profitability obtained from simulating the model under the following experiments: an absence of quality increases; a wage subsidy for firms aged 1 to 5; and an investment subsidy for firms aged 1 to 5. The first experiment helps shed light on the mechanisms in the model, while the subsidy experiments highlight the importance of developments during the early years of a firm.

5.4.1 No quality ladder

Panel A of Figure 6 presents the age profile of profitability obtained from simulating the model with the quality levels fixed at 1. As the figure indicates, mean profitability rises with age for the youngest firms and then declines steadily. The initial rise reflects the effect of survival, as the exit of young firms that received adverse shocks boosts the mean profitability of surviving firms. The figure, however, overstates the effect of survival on the age profile of profitability shown in Figure 5 as the exit rate in the simulated data increases sharply to 4.5% in the absence of quality increases. This reflects the fact that the potential growth opportunities associated with quality increases provide an important contribution to the continuation value of the firm, which determines exit. As noted before, wage pressures from new entrants drives down profits for mature firms.

Reflecting the absence of the quality ladder mechanism, profitability rises less, peaks earlier and declines at a faster rate than seen in Figure 5. The sharper rise in profitability for young firms in the baseline model reflects the quality increases obtained by them. The smaller decline in profitability for mid-aged and mature firms in the baseline model reflects the fact that these firms also obtain quality increases, albeit at a slower rate. Overall, these finding indicates that while the quality ladder mechanism plays an influential role in determining how profitability evolves with age, survival and vintage effects also have an impact.

Repeating the regressions reported in Tables 4 and 5 on the simulated data obtained without the quality ladder mechanism reveal that one obtains negative coefficients on age for the full sample. However, these effects are either much less negative or even positive when the sample is limited to firms younger than 19 years, the median age in the simulated data set with quality increases.[15] This reflects the fact that, in the absence of the quality ladder mechanism, age has countervailing effects for young firms. The youngest firms realize increases in profitability from survival, leading them to increase investment and sales growth. However, investment and sales growth quickly begin to decrease with age as the vintage effect dominates. The combination of these factors lead to modest and mixed age effects for young firms in the absence of the quality ladder mechanism. By contrast, as discussed before, the quality ladder mechanism implies that the early years are a period of rapid change for firms, leading to greater sensitivity of financing and growth decisions to age.

5.4.2 Wage subsidy for young firms

Panel B of Figure 6 plots the age profile of profitability obtained from simulating the model with an additional 5% wage subsidy for firms aged 1 to 5. While such direct wage subsidies are not often employed as a policy instrument, lower employment taxes for young firms can function as a wage subsidy by lowering labor costs.

As seen in the above figure, the wage subsidy directly increases profits for firms aged 1 to 5 by reducing labor costs. More importantly, it also bolsters investment in product development by these firms, as the higher profits translate to greater increases in value from a quality increase for young firms. This results in an increase in the average quality index from 7.30 to 7.46. In addition, the subsidy reduces the exit rate of firms from 2.76% to 2.71%, increases mean age from 25.8 to 26.1 years and increases average firm size by about 3.0%. This indicates that the increased quality levels arising from the initial wage subsidy for young firms benefits firms well after they stop receiving it, highlighting the important of developments in the early years of a firm's lifecycle.

[15]These results are available from the author upon request.

5.4.3 Investment subsidy for young firms

Panel C of Figure 6 plots the age profile of profitability obtained from simulating the model with an additional 5% investment subsidy for firms aged 1 to 5. Such an investment subsidy can reflect the effect of increased depreciation allowances for young firms.

The investment subsidy leads firms to increase their product development expenses, generating higher profits. This reflects the complementarity between physical investment and product development expenses in the model, as firms seek to expand their capacity and produce greater output following quality increases. The increased product development leads to an increase in the average quality index from 7.30 to 7.46, with corresponding reductions in exit rates and increases in mean age and size. As before, this indicates that the benefit firms receive from the initial investment subsidy continues well beyond the five years for which they receive it.

5.5 Implications for firm growth

In an influential study, Hsieh and Klenow (2014) find that manufacturing establishments in India and Mexico grow at a much slower rate than establishments in the U.S. They posit a number of possible explanations for this finding. One can use the framework in this study to examine whether endogenous product quality changes can generate divergence in firm growth across countries. Such a divergence may occur if robust legal systems or a high skilled workforce, which are much more likely to be prevalent in developed economies such as the U.S. or the U.K., are necessary for firms to successfully invest in improving their product quality.

Figure 7 plots the average sales of a firm from birth to 25 years of age. The solid line plots the growth obtained from the baseline model, while the dashed line plots the growth obtained from the model absent any quality improvements discussed in Section 5.4.1. The series are normalized such that, in the baseline model, the average sales of a firm upon entry equals 1. As the figure indicates, there is a substantial divergence in sales growth across the baseline model and the model without quality increases. This suggests that a stronger quality ladder mechanism may help explain the finding in Hsieh and Klenow (2014) that establishments in the U.S. growth much faster than those in India or Mexico.

The above growth rates are much larger than those documented in Hsieh and Klenow (2014). This reflects the fact that the model analysis examine sales growth at firms, whereas Hsieh and Klenow (2014) examine employment growth at establishments. Sales growth at firms may be much greater than employment growth at establishments as it encompasses growth in the number of establishments as well as quality increases that enables firms to charge higher prices for their products.

6 Regression evidence from U.K. firms

The model implies that the growth and financing of firms would be more sensitive to age for young firms, as seen by the regression results from the simulated data reported in Tables 4 and 5. This section examines these predictions using the data on U.K. firms. This test helps provide evidence on the quality ladder mechanism, as such a result does not arise in the simulations from the model without a quality ladder. This section also examines whether the estimated age effects are stronger in R&D-intensive industries, as the quality ladder mechanism would likely be stronger in such industries. In addition, it presents results from cross-sectional regressions using dummy variables for profitability jumps to examine whether young firms obtain quality increases at a faster rate than mature firms. Unfortunately, the data set does not include information on product development expenses such as research and development or advertising. As such, it is not possible to test this prediction directly.

6.1 Sales growth

Table 6 presents the results obtained from a two-stage Heckman selection model regression of sales growth on age, size and year and industry dummies.[16] The selection model corrects for survival bias in the data. This regression is carried out for all firms, firms grouped by age terciles, and for firms in R&D-intensive industries.[17] The first stage regression involves a probit regression of firm survival on age, size, profitability, cash holdings and year and industry dummies. This regression is carried out only for 2004 to 2008 as the sample excludes firms that exited prior to 2004. The estimates from this regression are used to calculate the inverse Mill's ratio for the entire sample. The second stage involves a panel regression of sales growth on age, size, inverse Mill's ratio and year and industry dummies. The exclusion restriction is that cash holdings influence exit but not sales growth. The reported standard errors are heteroskedasticity robust and adjust for clustering at the firm level.

[16]Specifically, the reported results are obtained from a random effects panel regression for sales growth with the Inverse Mill's ratio from a first-stage probit regression for survival as an additional control variable.

[17]The cut-off values for the age terciles equal 10 years and 9 months, and 23 years and 4 months.

The results indicate that younger firms have higher sales growth than older firms, similar to the finding in Cooley and Quadrini (2001). Strikingly, the effect of age on sales growth is much more pronounced for younger firms than for mature firms, as indicated by the regressions on the samples grouped by age. The regression coefficient for young firms is an order of magnitude greater than the coefficient for mature firms. This finding mirrors that obtained with the simulated data in Section 5.3, where the quality ladder mechanism results in sales growth being more sensitive to age for young firms. The results also indicate that the age coefficient is greater for firms in R&D-intensive industries, where a quality ladder mechanism may have a greater impact. The coefficient on the inverse Mill's ratio is statistically significant in some of the specifications, indicating the need to control for survival in the regression. In unreported results, the effect of age on sales growth (and other firm policies) remains stronger for young firms in the R&D-intensive industries.

The empirical results are robust to increasing the Winsorization threshold for outliers to 2.5 percent and to grouping firms by age into two bins based on the median age. However, the age coefficients become insignificant when one incorporates firm fixed effects. This reflects the fact that the data set spans only a small number of years, with much of the useful age variation in the data coming from the cross-section.

6.2 Investment

Table 7 presents the results obtained from a two-stage Heckman selection model regression of investment on age, size, sales growth, return on assets and year and industry dummies. As above, the first stage involves a probit regression of firm survival on the above regressors and cash holdings and the second stage involves a panel regression of investment on the above regressors and the inverse Mill's ratio from the first stage.[18] The reported standard errors are heteroskedasticity robust and adjust for clustering at the firm level.

The results also indicate that younger firms have higher investment rates, consistent with the model. As before, the coefficients on age vary strikingly across age groups, with much higher age coefficients for firms in the youngest tercile. In addition, firms in R&D-intensive industries have a stronger age effect than firms in other industries (this difference is statistically significant).

[18] The exclusion restriction that cash holdings have no effect on investment may not be suitable in this setting as cash holdings may impact investment. That said, the financing constraints literature that follows from Fazzari, Hubbard, and Petersen (1988) has typically focused on examining the relationship between cash flows and investment, rather than on the relationship between cash holdings and investment. One exception is Denis and Sibilkov (2010), who find a significant but modest effect of cash holdings on investment.

These findings are consistent with those obtained with the simulated data discussed in Section 5.3, providing evidence in favor of the model's implication that the sensitivity of firm growth to age is larger for young firms.

6.3 Financing

Table 8 presents the results obtained from panel logit regressions of external financing dummies on age, size, sales growth, return on assets and year and industry dummies. Panel A presents the results for the stock issuance dummy and Panel B presents the results for the broader external financing dummy. The reported standard errors are heteroskedasticity robust and adjust for clustering at the firm level.

The results reported in Panel A indicate that younger firms obtain equity financing more frequently. The effect of age on stock issuance is particularly pronounced for young firms, with the age coefficient becoming marginally positive for the mature firm subsample. As before, this finding reflects the differences in the age sensitivity of firms' financing decisions found in the simulated data. The results for the subsample of firms in R&D-intensive industries reveal a stronger age effect, consistent with the explanation that the age effect arises at least partially from the quality ladder mechanism.

The results for the broader external financing dummy variable, which equals one if a firm obtained either debt or equity financing during a year, follow those for the stock issuance dummy variable. However, they are somewhat less supportive of the model, as the age coefficient for young firms, while larger in magnitude, is statistically significant only at the 10 percent level. Also, one obtains similar age coefficients for the subsample of firms in the R&D-intensive industries and for the entire sample.

These results are obtained by definining dummy variables for when firms obtain external finance based on the growth of balance sheet measures of financing. The dummy variables used above equal 1 if equity or equity plus debt financing grew by more than 2 percent in a given year; changing this threshold to either 0.1 or 5 percent generates a broadly similar set of results.

6.4 Profitability jumps

The previous sections presented two findings that provide indirect support for the model. First, the effects of age on firms growth and financing decisions are much stronger for young firms than mature firms. Second, the age effects are stronger for firms in R&D-intensive industries. This section aims to provide a more direct test of the quality ladder mechanism in the model using a profitability jump dummy that attempts to capture firms that realized increases in product quality. As quality increases imply a permanent increase in profitability, one way of measuring whether a firm had realized a quality increase would be to look at the difference in mean return on assets across subperiods. A profitability jump dummy captures this idea by measuring whether the firm's average return on assets from 2004 to 2008 minus the firm's average return on assets over the preceding years exceeds a certain threshold. As this variable is defined using the available time series of data for each firm, it varies only in the cross-section.

Panel A of Table 9 presents the results of a cross-sectional regression of the profitability jump dummy on firms' median age, median size, volatility of profitability, and industry dummies. The threshold differences in mean return on assets for the dummy variable equals 10 percentage points. The reported standard errors are heteroskedasticity robust. The control variables include the realized volatility of profitability, as firms with more volatile profits will have a greater variation in differences in mean return on assets and therefore, by random chance, be more likely to be classified as having had a profitability jump. This would bias the age coefficients as young firms have more volatile profitability.

The findings indicate that young firms realize profitability jumps more often than mature firms. The full sample estimates from Panel A imply that a firm that is 10 years younger would have a 5 percentage point greater chance of realizing a profitability jumps. In comparison, about 15 percent of firms realized profitability jumps, indicating that the age effect is both statistically and economically significant. The relationship between age and profitability jumps arises only for firms in the lowest age tercile, consistent with the model where the quality ladder mechanism influences decisions mostly for young firms. Further, the coefficient on age for firms in R&D-intensive industries is greater than the corresponding coefficient for all firms, supportive of the model. The coefficient on the volatility of profitability is strongly positive in all the specifications, indicating the importance of controlling for volatility in this regression.

One concern is that the inclusion of realized volatility of profitability is not a sufficient control for the effect of volatility. As such, Panel B reports the results for a placebo regression of a profitability drop dummy that equals 1 if profitability declined by over 10 percentage points across the periods. While age has a negative effect in this regression, the negative effect arises from firms in the middle tercile, consistent with the decline in profitability for these firms shown in Figure 1.

In contrast to the profitability jump results, age has no effect on the youngest firms, suggesting that the profitability jump results for these firms are not merely reflecting a volatility effect. Also, age has no effect on profitability drops for firms in R&D-intensive industries.

In unreported results, one obtains the same qualitative patterns for the above regressions when the profitability jump and drop dummy variables are defined using a higher threshold value for profitability change of 20 percentage points.

7 Conclusion

This study documents that the profitability of firms follows a hump shape over the lifecycle. Profitability rises for young firms, peaks, and then declines slowly as firms mature. This finding is viewed through the lens of a dynamic lifecycle model, where firms invest in product development to increase product quality and face competition from new entrants who push up wages in the economy. When calibrated to data on public and private firms in the U.K., the model generates the hump-shaped age profile of profitability observed in the data.

Analysis of data obtained from simulating the model reveals that young firms grow faster and use more financing. Further, the effect of age on firms' policies are much stronger for young firms, reflecting the rapid changes in product quality for young firms in the model. Empirical tests using data on U.K. firms confirm these predictions, with the effect of age on firms' growth and financing decisions varying sharply across age terciles. In addition, firms in R&D-intensive industries have stronger age effects than firms in other industries, providing further support for the quality ladder mechanism.

Policy experiments using the model reveal that wage and investment subsidies for young firms generate substantial benefits that persist through the life of the firm. This arises due to the fact that the subsidies encourage additional product development expenditures that generate permanently higher quality levels. While much of the policy discussion on small firms have focused on providing financial support, these findings indicate that policy interventions such as payroll tax holidays and investment tax credits targeted toward young firms may yield substantial and persistent beneficial effects.

More generally, the model highlights the role of endogenous profitability changes on firm dynamics. For the most part, the existing literature on firm lifecycles typically consider profits as exogenously determined (for instance, see Cooley and Quadrini (2001), Cabral and Mata (2003), Miao (2005), Gamba and Triantis (2008) and Bolton, Chen, and Wang (2011)). Further research into understanding the effect of endogenous profitability changes on firms' decisions may prove fruitful.

Appendix

A Trend growth rates

Consider a steady state with an invariant distribution of firms with ages a_j and quality levels $q_j(n)$, and assume that the vintage productivity term μ_t grows at a constant rate g. This section shows that aggregate capital, consumption and wages also grows at g.

Differentiate (1) with respect to K_j to obtain the following:

$$\frac{\partial Y_j}{\partial K_j} = \left(\frac{K_j}{\mu_{t-a_j} L_j}\right)^{1-\alpha}.$$

Given a constant marginal product of capital, one obtains that the capital stock will be proportional to the effective labor input. I.e.,

$$K_j \propto \mu_{t-a_j} L_j.$$

Integrating over all firms, one obtains that

$$\begin{aligned} \int_j K_j dj &= s_0 \int_j \mu_{t-a_j} L_j dj, \\ \Rightarrow \qquad \int_j K_j dj &= s_0 \mu_t \int_j (1+g)^{-a_j} L_j dj, \end{aligned} \tag{A.1}$$

where s_0 is an integration constant and the second equation follows substituting in the vintage productivity terms of firms with the current vintage productivity term μ_t while adjusting for its constant growth rate. As the aggregate labor force is a constant, the distribution of labor remains invariant in the steady state. Thus, the integral on the L.H.S. of equation (A.1) equals a constant, implying that the aggregate capital stock grows at the same rate, g, as μ_t.

A similar derivation shows that the consumption aggregator C_t also grows at the constant rate g.

Applying the definition of the consumption aggregator and simplifying, one obtains the following:

$$\begin{aligned} C_t &= \left(\int_j q_j(n) Y_j^{1-\nu} dj\right)^{\frac{1}{1-\nu}}, \\ &= \left(\int_j q_j(n)\left(\mu_{t-a_j}^{1-\alpha} \tilde{z}_j K_j^{\alpha} L_j^{1-\alpha}\right)^{1-\nu} dj\right)^{\frac{1}{1-\nu}}, \\ &= \left(\int_j q_j(n)\left(\mu_t^{1-\alpha}(1+g)^{-a_j} \tilde{z}_j K_j^{\alpha} L_j^{1-\alpha}\right)^{1-\nu} dj\right)^{\frac{1}{1-\nu}}. \end{aligned}$$

In the steady state, the distribution of product quality, firm age, and labor inputs will remain invariant. As such, aggregate consumption will growth at the same rate as $\int_j \mu_t^{1-\alpha} K_j^{\alpha} dj$, which grows at rate g.

The growth rate of wages obtains from the first order condition for labor:

$$w_t L_j = q_j(n) C_t^{\nu}(1-\alpha)(1-\nu) Y_j^{1-\nu}.$$

Integrating over all firms in the economy, one obtains that

$$\begin{aligned} w_t \int_j L_j dj &= (1-\alpha)(1-\nu) C_t^{\nu} \int_j q_j(n) Y_j^{1-\nu} dj, \\ \Rightarrow \quad w_t L &= (1-\alpha)(1-\nu) C_t, \end{aligned}$$

implying that aggregate wages grow at the same rate, g, as aggregate consumption.

B Optimal product development

Proposition 1 *The first order condition for product development implies that*

$$\begin{aligned} \textit{marginal cost of funds} &= \textit{marginal benefit of product development expenses} \\ \Rightarrow \qquad 1 &= \beta(1+g) E_z \left[\frac{v(k', q(n+1), a+1, \tilde{z}') - v(k', q(n), a+1, \tilde{z}')}{q(n)}\right] b(1-p(r)). \end{aligned}$$

Proof.

Taking first order conditions from the above Bellman equation, one obtains that,

$$\text{marginal cost of funds} = \text{marginal benefit of product development expenses.}$$

In the absence of financial frictions, the marginal cost of funds (L.H.S of the above equation) equals one,

$$\text{L.H.S} = 1.$$

The marginal benefit of product development expenses (R.H.S. of the above equation) is given by:

$$\begin{aligned}
\text{R.H.S.} &= \frac{\partial}{\partial r}\left(\beta(1+g)E_z\left[p(r)v_c(k', q(n+1), a+1, \tilde{z}') + (1-p(r))v_c(k', q(n), a+1, \tilde{z}')\right]\right) \\
&= \beta(1+g)E_z\left[v(k', q(n+1), a+1, \tilde{z}') - v(k', q(n), a+1, \tilde{z}')\right]\frac{\partial p(r)}{\partial r}.
\end{aligned}$$

Some algebra yields that

$$\frac{\partial p(r)}{\partial r} = \frac{b}{q(n)}(1-p(r)).$$

Substituting this into the previous expressions completes the proof. ■

C Model solution and simulation

The optimal policies of the firm are obtained using value function iteration to solve the Bellman equation given in Equation (10). This process employs the optimal product development expense given in Proposition 1. At each step, the solution for physical investment is carried out numerically over a grid of values for capital. The numerical solution is obtained using the following grid sizes: a profitability shock grid with 5 values, a quality grid with values from 1 to 15, a capital grid with 120 values, and an age grid from 1 to 80. The simulated data sample is constructed using the value function solution and the associated optimal policy functions for financing and growth.

The simulated data set is obtained by simulating the model economy with 1000 firms over a period of 200 years. Observations in the first 100 years are discarded as a burn-in sample. An examination of the cross-sectional moments indicate that the simulations reach their steady state well before 100 years. Only a very small fraction of firms reach the maximum age level in the simulation. This simulated sample provides a steady state cross-section of firms that can be employed to further investigate firm policies in the model.

Firms exit endogenously in the simulation when their exit value exceeds the continuation value. Each firm that exits is replaced with a new firm of age 1 with capital stock near the bottom of it's grid and quality index $n = 1$. New entrants are assigned a random profitability shock level drawn from its unconditional distribution.

References

Acharya, Viral V., Stewart C. Myers, and Raghuram Rajan, 2011, The internal governance of firms, *Journal of Finance* 66, 689–720.

Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt, 2005, Competition and innovation: An inverted-U relationship, *Quarterly Journal of Economics* 120, 701–728.

Aghion, Philippe, Richard Blundell, Rachel Griffith, Peter Howitt, and Susanne Prantl, 2009, The effects of entry on incumbent innovation and productivity, *Review of Economics and Statistics* 91, 20–32.

Aghion, Philippe, and Peter Howitt, 1992, A model of growth through creative destruction, *Econometrica* 60, 323–351.

Albuquerque, Rui, and Hugo A. Hopenhayn, 2004, Optimal lending contracts and firm dynamics, *Review of Economic Studies* 71, 285–315.

Angelini, Paolo, and Andrea Generale, 2008, On the evolution of firm size distributions, *American Economic Review* 98, 426–438.

Bitler, Marianne P., Tobias J. Moskowitz, and Annette Vissing-Jorgensen, 2005, Testing agency theory with entrepreneur effort and wealth, *Journal of Finance* 60, 539–576.

Bloom, Nick, Stephen Bond, and John van Reenen, 2007, Uncertainty and investment dynamics, *Review of Economic Studies* 74, 391–415.

Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of Tobin's Q, corporate investment, financing, and risk management, *Journal of Finance* 66, 1545–1578.

Brav, Omer, 2009, Access to capital, capital structure, and the funding of the firm, *Journal of Finance* 64, 263–308.

Cabral, Luis M. B., and Jose Mata, 2003, On the evolution of the firm size distribution: Facts and theory, *American Economic Review* 93, 1075–1090.

Cagetti, Marco, and Mariacristina De Nardi, 2006, Entrepreneurship, frictions, and wealth, *Journal of Political Economy* 114, 835–870.

Chen, Hui, Jianjun Miao, and Neng Wang, 2010, Entrepreneurial finance and nondiversifiable risk, *Review of Financial Studies* 23, 4348–4388.

Clementi, Gian Luca, and Berardino Palazzo, 2015, Entry, exit, firm dynamics, and aggregate fluctuations, forthcoming, *American Economic Journal: Macroeconomics.*

Cooley, Thomas F., and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.

Cooper, Russell W., and John Haltiwanger, 2006, On the nature of capital adjustment costs, *Review of Economic Studies* 73, 611–633.

Cunha, Flavio, and James Heckman, 2007, The technology of skill formation, *American Economic Association (Papers & Proceedings)* 97, 31–47.

DeAngelo, Harry, Linda DeAngelo, and Rene M. Stulz, 2006, Dividend policy and the earned/contributed capital mix: A test of the lifecycle theory, *Journal of Financial Economics* 81, 293–315.

Denis, David J., and Valeriy Sibilkov, 2010, Financial constraints, investment, and the value of cash holdings, *Review of Financial Studies* 23, 247–269.

Dinopoulos, Elias, and Peter Thompson, 1998, Schumpeterian growth without scale effects, *Journal of Economic Growth* 3, 313–335.

Dunne, Timothy, Mark J. Roberts, and Larry Samuelson, 1989, The growth and failure of U.S. manufacturing sector, *Quarterly Journal of Economics* 104, 671–98.

Eberly, Janice, Sergio Rebelo, and Nicolas Vincent, 2008, Investment and value: A neoclassical benchmark, NBER working paper #13866.

Ericson, Richard, and Ariel Pakes, 1995, Markov-perfect industry dynamics: A framework for empirical work, *Review of Economic Studies* 62, 53–82.

Fazzari, Steven M., R. Glenn Hubbard, and Bruce C. Petersen, 1988, Financing constraints and corporate investment, *Brookings Papers on Economic Activity* 1, 141–195.

Foster, Lucia, John Haltiwanger, and Chad Syverson, 2008, Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?, *American Economic Review* 98, 394–425.

Foster, Lucia, John Haltiwanger, and Chad Syverson, 2012, The slow growth of new plants: Learning about demand?, NBER working paper #17853.

Gamba, Andrea, and Alexander J. Triantis, 2008, The value of financial flexibility, *Journal of Finance* 63, 2263–2296.

Gomes, João F., 2001, Financing investment, *American Economic Review* 90, 1263–1285.

Grossman, Gene M., and Elhanan Helpman, 1991, Quality ladders and product cycles, *Quarterly Journal of Economics* 106, 557–586.

Haltiwanger, John C., Ron S. Jarmin, and Javier Miranda, 2013, Who creates jobs? Small vs large vs young, *Review of Economics and Statistics* 95, 347–361.

Harford, Jarrad, 2005, What drives merger waves?, *Journal of Financial Economics* 77, 529–560.

Hayashi, Fumio, 1982, Tobin's marginal q and average q: A neoclassical interpretation, *Econometrica* 50, 213–224.

Heckman, James J., 1979, Sample selection bias as a specification error, *Econometrica* 47, 153–161.

Hennessy, Christopher A., and Toni M. Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.

Hopenhayn, Hugo A., 1992, Entry, exit, and firm dynamics in long run equilibrium, *Econometrica* 60, 1127–1150.

Hsieh, Chang-Tai, and Peter J. Klenow, 2014, The lifecycle of plants in India and Mexico, *Quarterly Journal of Economics* 129, 1035–1084.

Huynh, Kim P., and Robert J. Petrunia, 2010, Age effects, leverage and firm growth, *Journal of Economic Dynamics & Control* 34, 1003–1013.

Huynh, Kim P., and Robert J. Petrunia, 2015, Post-entry struggle for life and pre-exit shadow of death from a financial perspective, Forthcoming, *International Journal of the Economics of Business*.

Jovanovic, Boyan, 1982, Selection and the evolution of industry, *Econometrica* 50, 649–670.

Jovanovic, Boyan, and Peter Rousseau, 2008, Mergers as reallocation, *Review of Economics and Statistics* 90, 765–776.

Klette, Tor Jacob, and Samuel Kortum, 2004, Innovating firms and aggregate innovation, *Journal of Political Economy* 112, 986–1018.

Lee, Yoonsoo, and Toshihiko Mukoyama, 2015, Entry and exit of manufacturing plants over the business cycle, *European Economic Review* 77, 20–27.

Lentz, Rasmus, and Dale T. Mortensen, 2008, An empirical model of growth through innovation, *Econometrica* 76, 1317–1373.

Lucas, Robert E., Jr., 1978, On the size distribution of business firms, *Bell Journal of Economics* 9, 508–523.

Luttmer, Erzo G.J., 2011, On the mechanics of firm growth, *Review of Economic Studies* 78, 1042–1068.

Miao, Jianjun, 2005, Optimal capital structure and industry dynamics, *Journal of Finance* 60, 2621–2660.

Moskowitz, Tobias J., and Annette Vissing-Jorgensen, 2002, The returns to entrepreneurial investment: A private equity premium puzzle?, *American Economic Review* 92, 745–778.

Moyen, Nathalie, 2004, Investment-cash flow sensitivities: Constrained versus unconstrained firms, *Journal of Finance* 59, 2061–2092.

Rajan, Raghuram G., and Luigi Zingales, 1998, Financial dependence and growth, *American Economic Review* 88, 559–586.

Solow, Robert M., James Tobin, Carl C. von Weizsäcker, and Menahem E. Yaari, 1966, Neoclassical growth with fixed factor proportions, *Review of Economic Studies* 33, 79–115.

Warusawitharana, Missaka, 2008, Corporate asset purchases and sales: Theory and evidence, *Journal of Financial Economics* 87, 471–497.

Warusawitharana, Missaka, 2015, Research and development, profits and firm value: A structural estimation, *Quantitative Economics* 6, 531–565.

Whited, Toni M., 2006, External finance constraints and the intertemporal pattern of intermittent investment, *Journal of Financial Economics* 81, 467–502.

Table 1: Summary statistics

The table presents summary statistics for variables of interest from the Amadeus data set. The table reports means and standard deviations (denoted Std.) for all firms, firms grouped by age terciles, and firms in R&D-intensive industries. Section 2 details the construction of the sample. Age is measured from the date of incorporation. Sales growth equals the annual growth rate of revenue. Return on assets equals the firm's operating income divided by average total assets. Investment equals the growth rate of fixed assets. The dummy variables are defined as follows. Exit equals 1 if the firm did not have a legal status of "active". Stock issuance equals 1 if the firms contributed equity is greater than the corresponding value for the previous year plus 2 percent. External financing equals 1 if the sum of book debt and contributed equity is greater than the corresponding sum for the previous year plus 2 percent. Profitability jump equals 1 if the firm's return on assets over the last five years of the sample minus the firm's return on assets over the first five years of the sample was greater than 10 percentage points; profitability drop jummy equals one if the corresponding difference was less than minus 10 percentage points. All continuous variables (except age) are Winsorized at the 99th percentile to reduce the impact of outliers. Return on assets, sales growth, investment, and dummies on exit, stock issuance, external finance, and profitability jump and drop are reported in percent terms.

	All firms		Grouped by age terciles						High R&D firms	
			Young		Mid-aged		Mature			
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Age	22.9	21.2	6.3	2.4	16.4	3.6	46.0	21.8	18.6	18.2
Log assets	15.6	1.8	15.3	1.8	15.6	1.7	16.1	1.7	15.5	1.7
Return on assets	9.2%	22.0%	9.2%	28.6%	9.9%	20.0%	8.4%	15.2%	8.5%	27.8%
Growth rates of										
Sales	20.0%	72.3%	38.8%	102.2%	12.8%	52.5%	8.7%	44.7%	24.8%	80.4%
Fixed assets	16.5%	90.7%	25.7%	110.8%	14.3%	86.5%	10.0%	70.9%	20.3%	105.2%
Dummy variables for										
Exit	1.8%	13.4%	2.7%	16.3%	1.6%	12.6%	1.1%	10.3%	1.6%	12.6%
Stock issuance	5.8%	23.4%	9.3%	29.1%	4.6%	21.0%	3.8%	19.1%	8.2%	27.4%
External finance	47.3%	49.9%	49.4%	50.0%	46.7%	49.9%	46.0%	49.8%	48.1%	50.0%
Profitability jump	14.1%	34.8%	20.2%	40.1%	13.9%	34.6%	8.9%	28.6%	19.3%	40.9%
Profitability drop	17.3%	37.9%	21.2%	40.8%	18.2%	38.6%	13.1%	33.8%	21.2%	39.4%

Table 2: Calibrated moments

The table reports the moments used in calibrating the model parameters. The second column reports the moments from the Amadeus data set while the third column reports the moments from the data set obtained by simulating the model. Section 2 details the construction of the sample and the variable definitions of the Amadeus data set while Section 5.1 details the construction of the simulated data set.

Moment	Data	Model
Mean profits	0.092	0.088
Std. dev. of profits	0.220	0.153
Mean profits at entry	0.077	0.073
Mean profits at peak	0.106	0.112
Mean profits of mature firms	0.084	0.074
Exit rate	0.018	0.028
Mean investment	0.165	0.190
Mean age	22.9	25.8
25^{th} percentile of age	8.2	9.0
75^{th} percentile of age	29.5	39.0

Table 3: Estimates from U.K. data – Exit

The table reports the results obtained from a logit regression of exit on age and control variables. The regression is carried out on data from 2004 onwards. The table reports results for all firms and firms grouped by age terciles. Section 2 details the construction of the sample and the variable definitions. A firm is considered to have exited if its legal status was not classified as "active". All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively. Return on assets and sales growth are measured in percent terms.

	All firms	Grouped by age		
		Young	Mid-aged	Mature
Age	-0.014^{**}	-0.043^{**}	-0.008	0.002
	(0.002)	(0.012)	(0.009)	(0.002)
Size	-0.23^{**}	-0.19^{**}	-0.24^{**}	-0.28^{**}
	(0.01)	(0.02)	(0.02)	(0.03)
Profitability	-1.06^{**}	-0.79^{**}	-1.42^{**}	-1.81^{**}
	(0.06)	(0.07)	(0.11)	(0.16)
Sales growth	-0.00	-0.02	0.01	-0.40
	(0.03)	(0.03)	(0.09)	(0.27)
Cash holdings	-1.18^{**}	-1.37^{**}	-0.77^{**}	-1.21^{**}
	(0.14)	(0.20)	(0.24)	(0.41)
Observations	165781	55611	55173	52724
Pseudo R-squared	0.09	0.08	0.10	0.09

Table 4: Estimates from simulated data – Firm growth

Panels A and B, respectively, report the results obtained from estimating panel regressions for sales growth and investment on the simulated data set. The results are reported for all firms, and for firms split by age groups. Section 5.1 details the construction of the simulated data set. The simulation sample excludes new entrants. The standard errors are heteroskedasticity robust. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively. Sales growth, return on assets and investment are measured in percent terms.

Panel A: Sales growth

	All firms	Grouped by age	
		< median	>= median
Age	−0.50**	−2.11**	−0.14**
	(0.03)	(0.18)	(0.05)
Size	−5.44**	−5.88**	4.45**
	(0.44)	(0.85)	(0.67)
Observations	98310	48925	49385
Pseudo R-squared	0.00	0.01	0.00

Panel B: Investment

	All firms	Grouped by age	
		< median	>= median
Age	−0.31**	−2.35**	−0.07**
	(0.00)	(0.02)	(0.00)
Size	−6.74**	−1.39**	−1.77**
	(0.05)	(0.11)	(0.05)
Profitability	0.74**	0.77**	0.54**
	(0.01)	(0.01)	(0.01)
Sales growth	−0.001**	−0.002**	−0.003**
	(0.000)	(0.000)	(0.000)
Observations	98310	48925	49385
Pseudo R-squared	0.59	0.62	0.61

Table 5: Estimates from simulated data – Quality increases and financing

Panels A and B, respectively, report the results obtained from estimating logit regressions for external financing and quality increases on the simulated data set. The results are reported for all firms and for firms split by age groups. Section 5.1 details the construction of the simulated data set. The simulation sample excludes new entrants. The standard errors are heteroskedasticity robust. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively.

Panel A: External financing

	All firms	Grouped by age	
		< median	>= median
Age	-0.06^{**}	-0.19^{**}	-0.04^{**}
	(0.001)	(0.003)	(0.001)
Size	-0.36^{**}	0.28	-0.45^{**}
	(0.01)	(0.01)	(0.01)
Return on assets	-5.81^{**}	-6.18^{**}	-12.41^{**}
	(0.12)	(0.13)	(0.62)
Sales growth	0.08^{**}	0.09^{**}	0.05^{**}
	(0.00)	(0.01)	(0.00)
Observations	98310	48925	49385
Pseudo R-squared	0.31	0.22	0.15

Panel B: Quality increases

	All firms	Grouped by age	
		< median	>= median
Age	-0.077^{**}	-0.261^{**}	-0.028^{**}
	(0.002)	(0.004)	(0.001)
Size	0.54^{**}	1.29^{**}	0.26^{**}
	(0.01)	(0.02)	(0.01)
Observations	98310	48925	49385
Pseudo R-squared	0.12	0.16	0.04

Table 6: Estimates from U.K. data – Sales growth

The table reports the results obtained from regressing sales growth on age and control variables using a Heckman two-step approach to account for survival. The table reports results for all firms, firms grouped by age terciles, and firms in R&D-intensive industries. Section 2 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively. Sales growth is measured in percent terms.

	All firms	Grouped by age			R&D-intensive
		Young	Mid-aged	Mature	industries
Age	-0.66^{**}	-10.86^{**}	-0.60^{**}	-0.07^{**}	-0.97^{**}
	(0.01)	(0.18)	(0.05)	(0.01)	(0.03)
Size	1.82^{**}	2.01^{**}	0.88^{**}	0.88^{**}	2.55^{**}
	(0.19)	(0.31)	(0.19)	(0.17)	(0.30)
Inverse Mill's ratio	-0.11	0.82^{**}	-1.43^{**}	-1.30^{**}	-0.07
	(0.14)	(0.22)	(0.14)	(0.17)	(0.30)
Observations	284786	93537	95250	95999	74838
Pseudo R-squared	0.03	0.07	0.01	0.01	0.03

Table 7: Estimates from U.K. data – Investment

The table reports the results obtained from regressing investment on age and control variables using a Heckman two-step approach to account for survival. The table reports results for all firms, firms grouped by age terciles, and firms in R&D-intensive industries. Section 2 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively. Return on assets and sales growth are measured in percent terms.

	All firms	Grouped by age			R&D-intensive industries
		Young	Mid-aged	Mature	
Age	-0.19^{**}	-0.58^{*}	-0.40^{**}	-0.08^{**}	-0.23^{**}
	(0.01)	(0.18)	(0.08)	(0.01)	(0.02)
Size	3.84^{**}	3.23^{**}	3.22^{**}	2.84^{**}	5.62^{**}
	(0.15)	(0.30)	(0.22)	(0.19)	(0.34)
Profitability	0.11^{**}	0.11^{**}	0.09^{**}	0.07^{**}	0.11^{**}
	(0.01)	(0.02)	(0.02)	(0.03)	(0.02)
Sales growth	0.18^{**}	0.18^{**}	0.20^{**}	0.16^{**}	0.22^{**}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Inverse Mill's ratio	0.57^{**}	0.55^{*}	0.25	-0.13	1.11^{**}
	(0.13)	(0.22)	(0.19)	(0.21)	(0.29)
Observations	273480	86975	91903	94602	70993
Pseudo R-squared	0.03	0.03	0.02	0.02	0.04

Table 8: Estimates from U.K. data – Financing

Panels A and B, respectively, report the results obtained from logistic regressions of stock issuance and external financing dummies on age and control variables. Stock issuance equals 1 if the firms contributed equity is greater than the corresponding value for the previous year plus 2 percent. External financing equals 1 if the sum of book debt and contributed equity is greater than the corresponding sum for the previous year plus 2 percent. The table reports results for all firms, firms grouped by age terciles, and firms in R&D-intensive industries. Section 2 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. $^{+}$, * and ** denote statistical significance at the 10, 5 and 1 percent confidence levels, respectively. Sales growth and return on assets are reported in percent terms.

Panel A: Stock issuance

	All firms	Grouped by age			R&D-intensive
		Young	Mid-aged	Mature	industries
Age	-0.014^{**}	-0.086^{**}	-0.036^{**}	0.002^{*}	-0.020^{**}
	(0.001)	(0.006)	(0.005)	(0.001)	(0.002)
Size	0.20^{**}	0.18^{**}	0.23^{**}	0.25^{**}	0.22^{**}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Return on assets	-1.56^{**}	-1.39^{**}	-1.51^{**}	-1.89^{**}	-1.47^{**}
	(0.04)	(0.05)	(0.08)	(0.12)	(0.06)
Sales growth	0.29^{**}	0.21^{**}	0.30^{**}	0.29^{**}	0.27^{**}
	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)
Observations	276167	85508	92785	97683	71366
Pseudo R-squared	0.08	0.09	0.06	0.06	0.08

Panel B: External financing

	All firms	Grouped by age			R&D-intensive
		Young	Mid-aged	Mature	industries
Age	-0.003^{**}	-0.006^{+}	-0.004^{+}	-0.003^{**}	-0.003^{**}
	(0.000)	(0.004)	(0.002)	(0.000)	(0.001)
Size	0.08^{**}	0.04^{**}	0.09^{**}	0.11^{**}	0.08^{**}
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Return on assets	-1.39^{**}	-1.45^{**}	-1.44^{**}	-1.33^{**}	-1.28^{**}
	(0.03)	(0.04)	(0.06)	(0.07)	(0.05)
Sales growth	0.27^{**}	0.21^{**}	0.35^{**}	0.34^{**}	0.28^{**}
	(0.01)	(0.01)	(0.03)	(0.03)	(0.02)
Observations	208078	64339	68241	75498	49231
Pseudo R-squared	0.02	0.03	0.02	0.01	0.02

Table 9: Estimates from U.K. data – Profitability jumps and drops

Panels A and B, respectively, report the results obtained from estimating cross-sectional logit regressions of profitability jump and drop dummies on median age and controls. The profitability jump dummy variable equals one if a firm's average return on assets from 2004 to 2008 minus its average return on assets from before 2004 was greater than 10 percentage points. The profitability drop dummy equals one if the above difference was less than minus 10 percentage points. The table reports results for all firms, firms grouped by age terciles, and firms in R&D-intensive industries. Section 2 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust. * and ** denote statistical significance at the 5 and 1 percent confidence levels, respectively.

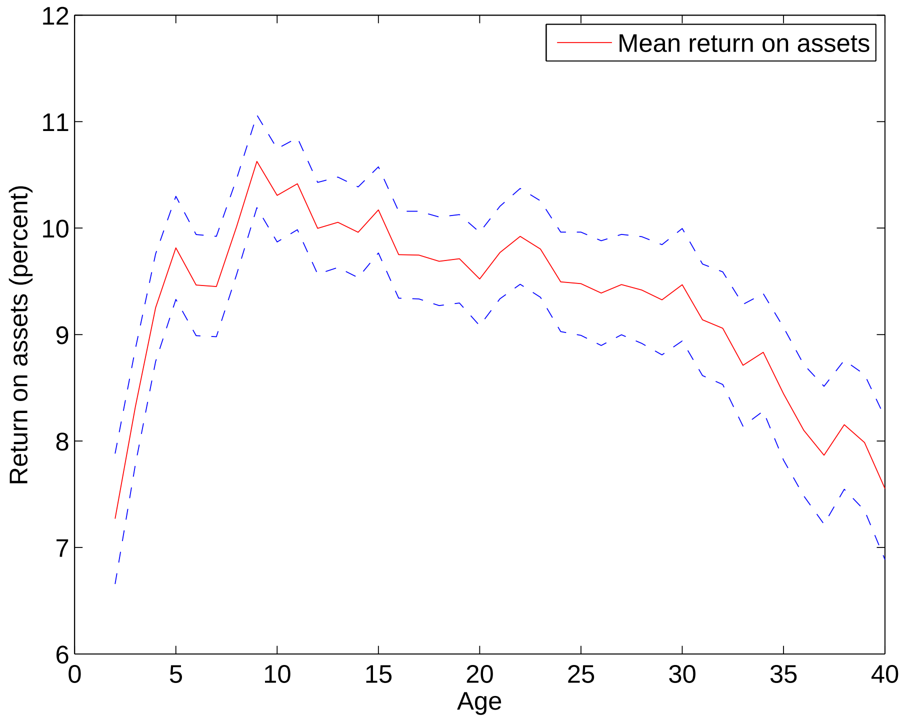
Panel A: Profitability jumps

	All firms	Grouped by age			R&D-intensive industries
		Young	Mid-aged	Mature	
Median age	-0.005^{**}	-0.042^{**}	0.007	0.002	-0.007^{**}
	(0.001)	(0.013)	(0.009)	(0.002)	(0.002)
Median size	-0.107^{**}	-0.038^{*}	-0.165^{**}	-0.163^{**}	-0.109^{**}
	(0.013)	(0.018)	(0.023)	(0.028)	(0.022)
Std. dev. profitability	8.385^{**}	7.289^{**}	9.179^{**}	10.542^{**}	7.454^{**}
	(0.217)	(0.280)	(0.391)	(0.589)	(0.308)
Observations	33459	10843	11234	11238	8684
Pseudo R-squared	0.19	0.18	0.19	0.16	0.18

Panel B: Profitability drops

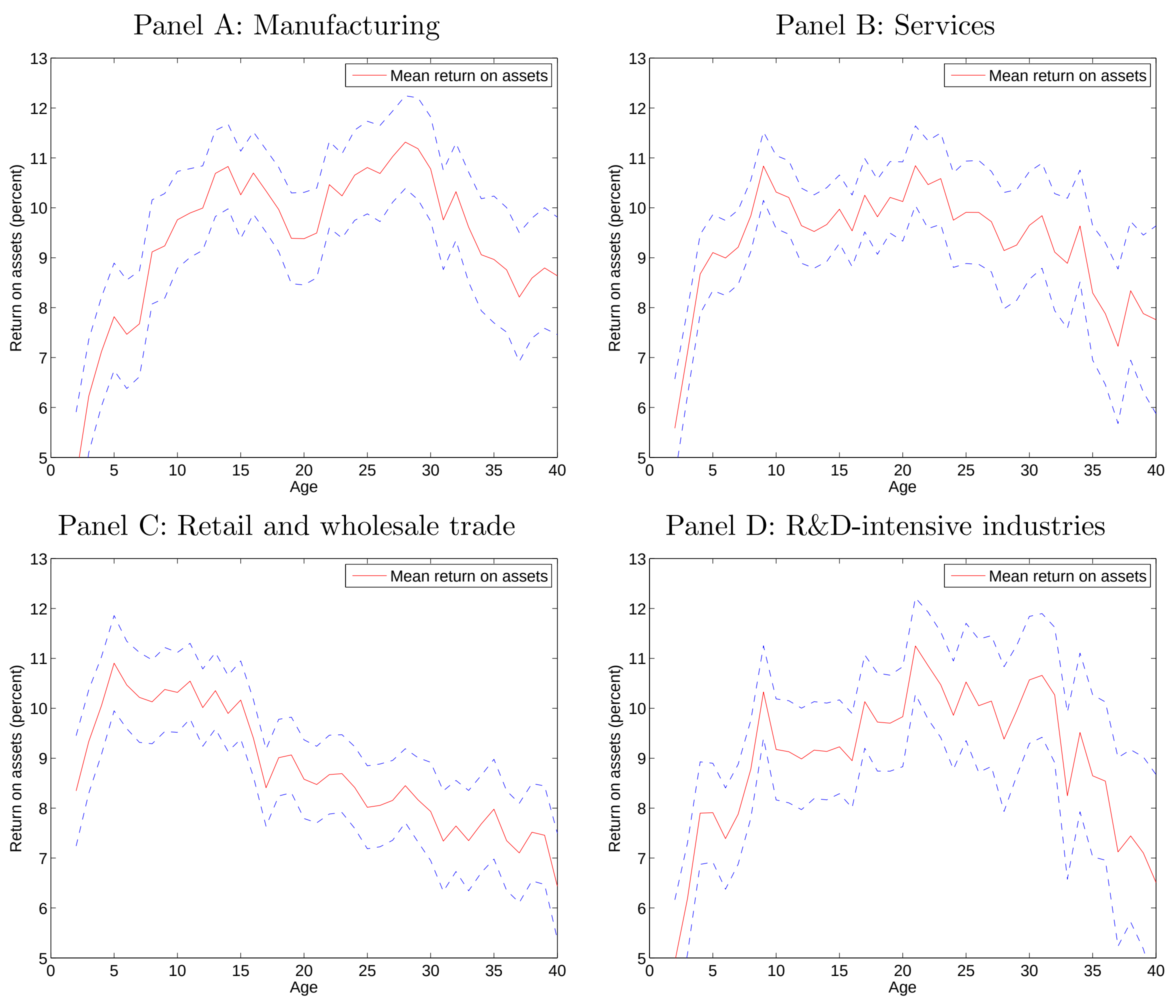
	All firms	Grouped by age			R&D-intensive industries
		Young	Mid-aged	Mature	
Median age	-0.004^{**}	0.022	-0.020^{**}	-0.001	-0.002
	(0.001)	(0.012)	(0.008)	(0.002)	(0.002)
Median size	-0.053^{**}	-0.157^{**}	0.002	0.059^{**}	-0.050^{**}
	(0.010)	(0.016)	(0.017)	(0.019)	(0.017)
Std. dev. profitability	5.552^{**}	3.056^{**}	6.771^{**}	14.457^{**}	3.214^{**}
	(0.194)	(0.187)	(0.387)	(0.711)	(0.214)
Observations	33456	10822	11264	11296	8684
Pseudo R-squared	0.09	0.07	0.10	0.20	0.05

Figure 1: Age profile of profitability



The figure plots mean profitability level as a function of age. Profitability (return on assets) is defined as operating income before depreciation scaled by average total assets. The solid (red) line plots the mean profitability level while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 2 details the construction of the sample using the Amadeus data set. Age is measured from the year of incorporation. The sample includes firms aged 2 to 40.

Figure 2: Age profile of profitability – Major industries



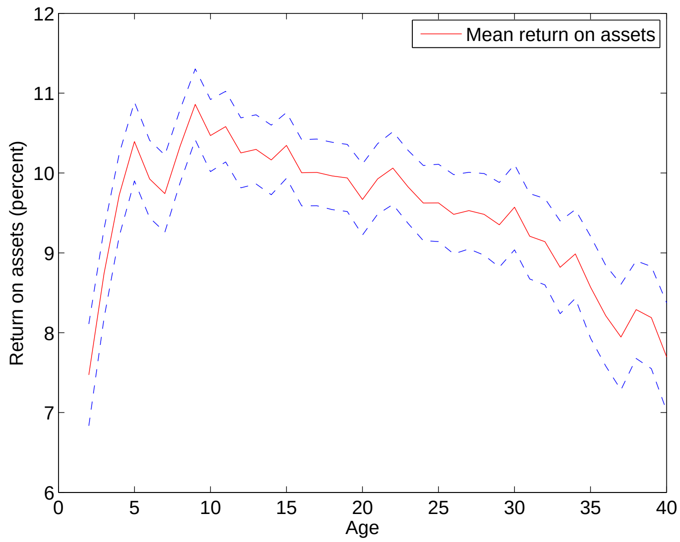
The figure plots mean profitability level as a function of age for firms in selected sectors. Panels A, B, C, and D, respectively, plot the age profile of profitability for firms in manufacturing, services, wholesale and retail trade and R&D-intensive industries. Firms are classified into major industry groups based on their 2-digit SIC codes. Profitability (return on assets) is defined as operating income before depreciation scaled by average total assets. The solid (red) line plots the mean profitability level while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 2 details the construction of the sample using the Amadeus data set. Age is measured from the year of incorporation. The sample includes firms aged 2 to 40.

Figure 3: Age profile of profitability changes

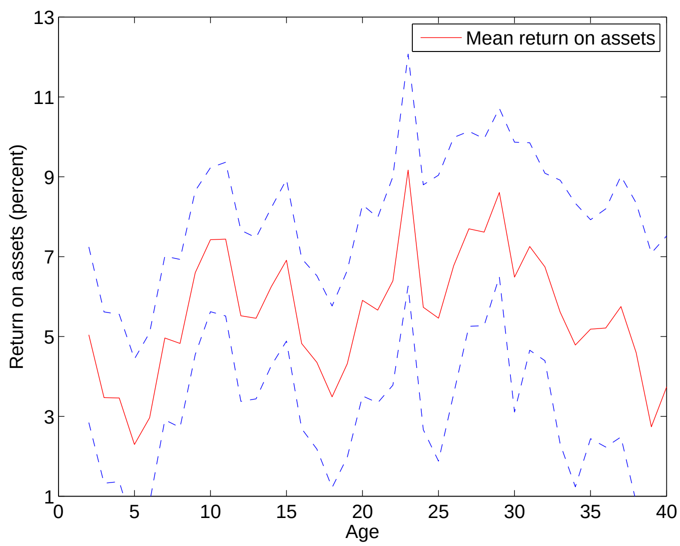
The figure plots the mean change in profitability from age t to $t+1$ as a function of age, t. Profitability (return on assets) is defined as operating income before depreciation scaled by average total assets. The solid (red) line plots the mean profitability change while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 2 details the construction of the sample using the Amadeus data set. Age is measured from the year of incorporation. The sample includes firms aged 2 to 40.

Figure 4: Age profile of profitability – by survival

Panel A: Surviving firms

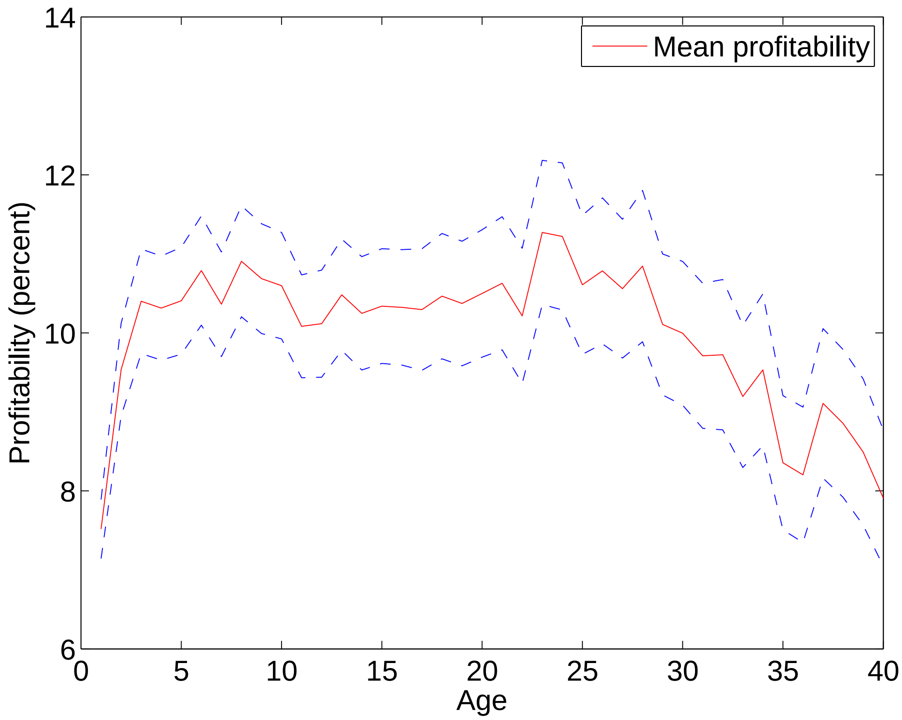


Panel B: Exiting firms



Panels A and B, respectively, plot the mean profitability level as a function of age for firms that survive until the end of the sample and for those who exit in between. Profitability (return on assets) is defined as operating income before depreciation scaled by average total assets. A firm is considered to have survived if it's legal status is active as of the last reporting period. The solid (red) line plots the mean profitability level while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 2 details the construction of the sample using the Amadeus data set. Age is measured from the year of incorporation. The sample includes firms aged 2 to 40.

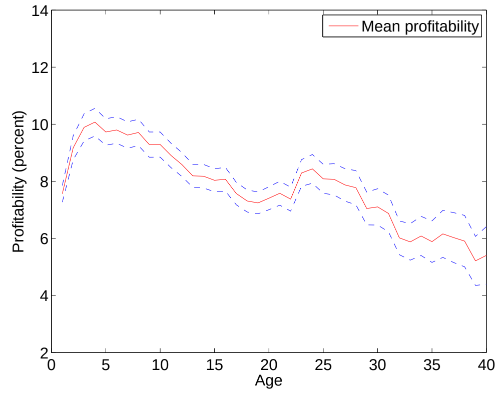
Figure 5: Simulated age profile of profitability



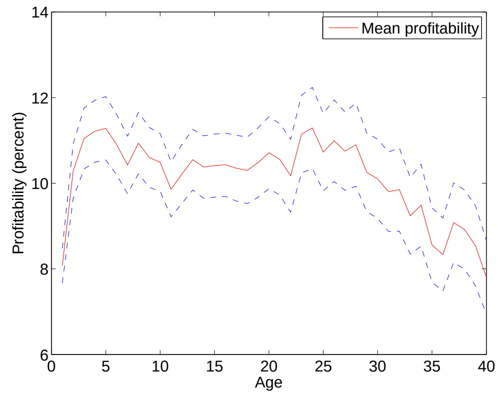
Using the simulated data set, the figure plots the mean profitability level as a function of age. The solid (red) line plots the mean profitability level while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 5.1 and Appendix C detail the construction of the simulated data set. The sample includes firms aged 1 to 40.

Figure 6: Simulated age profile of profitability – experiments

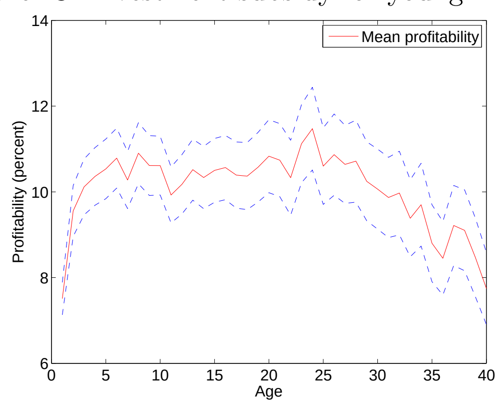
Panel A: No quality ladder



Panel B: Wage subsidy for young firms

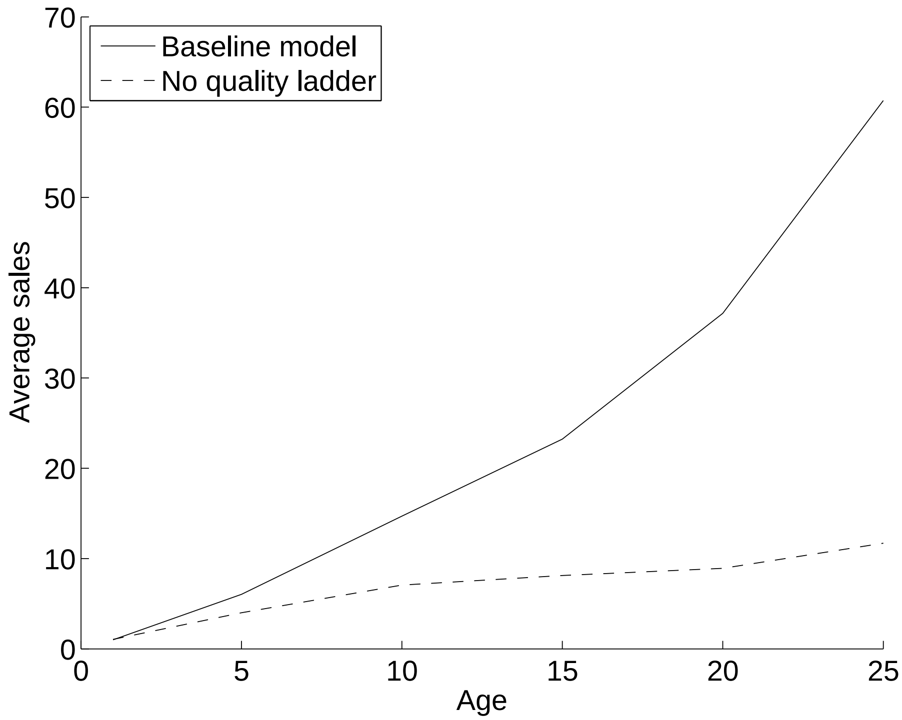


Panel C: Investment subsidy for young firms



The figure plots the simulated age profiles of profitability from a number of model experiments. Panel A turns off the quality ladder mechanism; Panel B examines a wage subsidy equal to 5 percent of the wage bill for firms aged 1 to 5; and Panel C examines an investment subsidy of 5 percent for firms aged 1 to 5. Figure 5 presents the comparable age profile from the baseline model. The solid (red) line plots the mean profitability level while the dashed (blue) lines plot the associated 95 percent confidence intervals. Section 5.1 and Appendix C detail the construction of the simulated data set. The sample includes firms aged 1 to 40.

Figure 7: Implications for firm growth



The figure plots the mean revenues for a firm as a function of its age. The solid line plots mean revenues obtained by simulating the baseline model discussed in Section 5.1; the dashed line plots mean revenues obtained from the model without the quality ladder mechanism discussed in Section 5.4.1. Series are normalized such that the mean revenues of a firm upon entry in the baseline model equals 1.